Pinning Down versus Density

Lajos Soukup

Alfréd Rényi Institute of Mathematics
Hungarian Academy of Sciences

http://www.renyi.hu/∼soukup

Twelfth Symposium on General Topology
and its Relations to Modern Analysis and Algebra

joint work with I. Juhász, J. van Mill and Z. Szentmiklóssy

Cardinal functions

Cardinal functions

$$X \mapsto F(X) \in Card$$

$$X \approx Y \Longrightarrow F(X) = F(Y)$$

Cardinal functions

$$X \mapsto F(X) \in Card$$

$$X \approx Y \Longrightarrow F(X) = F(Y)$$

- $|X|$

Cardinal functions

$$X \mapsto F(X) \in Card$$

$$X \approx Y \Longrightarrow F(X) = F(Y)$$

- $|X|$
- $w(X) = \min\{|\mathcal{B}| : \mathcal{B} \text{ is a base}\}$

Cardinal functions

$$X \mapsto F(X) \in Card$$

$$X \approx Y \Longrightarrow F(X) = F(Y)$$

- $|X|$
- $w(X)$= $\min\{|\mathcal{B}| : \mathcal{B} \text{ is a base}\}$
- $d(X)$= $\min\{|D| : D \subset^{dense} X\}$

Basic inequalities

- $|X|$
- $\mathrm{w}(X) = \min\{|\mathcal{B}| : \mathcal{B} \text{ is a base}\}$
- $\mathrm{d}(X) = \min\{|D| : D \subset^{dense} X\}$

Basic inequalities

- $|X|$
- $\mathrm{w}(X) = \min\{|\mathcal{B}| : \mathcal{B} \text{ is a base}\}$
- $\mathrm{d}(X) = \min\{|D| : D \subset^{dense} X\}$

- $\mathrm{d}(X) \leq \mathrm{w}(X) \leq 2^{|X|}$.

Basic inequalities

- $|X|$
- $\mathrm{w}(X) = \min\{|\mathcal{B}| : \mathcal{B} \text{ is a base}\}$
- $\mathrm{d}(X) = \min\{|D| : D \subset^{dense} X\}$

- $\mathrm{d}(X) \leq \mathrm{w}(X) \leq 2^{|X|}$.
- (Posposil) X Hausdorff: $|X| \leq 2^{2^{\mathrm{d}(X)}}$. Sharp: $\beta\omega$

Basic inequalities

- $|X|$
- $\mathrm{w}(X) = \min\{|\mathcal{B}| : \mathcal{B} \text{ is a base}\}$
- $\mathrm{d}(X) = \min\{|D| : D \subset^{\mathit{dense}} X\}$

- $\mathrm{d}(X) \leq \mathrm{w}(X) \leq 2^{|X|}$.
- (Posposil) X Hausdorff: $|X| \leq 2^{2^{\mathrm{d}(X)}}$. Sharp: $\beta\omega$
- X regular: $\mathrm{w}(X) \leq 2^{\mathrm{d}(X)}$.

Basic inequalities

- $|X|$
- $\mathrm{w}(X) = \min\{|\mathcal{B}| : \mathcal{B} \text{ is a base}\}$
- $\mathrm{d}(X) = \min\{|D| : D \subset^{dense} X\}$

- $\mathrm{d}(X) \leq \mathrm{w}(X) \leq 2^{|X|}$.
- (Posposil) X Hausdorff: $|X| \leq 2^{2^{\mathrm{d}(X)}}$. Sharp: $\beta\omega$
- X regular: $\mathrm{w}(X) \leq 2^{\mathrm{d}(X)}$.
- X Hausdorff: $\mathrm{w}(X) \leq 2^{2^{2^{d(X)}}}$. Sharp (Kunen - Juhász)

Pinning down number

Pinning down number

$U : X \to \tau_X$ is a neighborhood assignment on X iff

$$p \in U(p) \text{ for all } p \in X$$

Pinning down number

$U : X \to \tau_X$ is a neighborhood assignment on X iff

$$p \in U(p) \text{ for all } p \in X$$

$A \subset X$ pins down a neighborhood assignment U iff

$$A \cap U(p) \neq \emptyset \text{ for all } p \in X$$

Pinning down number

$U : X \to \tau_X$ is a **neighborhood assignment** on X iff

$$p \in U(p) \text{ for all } p \in X$$

$A \subset X$ **pins down** a neighborhood assignment U iff

$$A \cap U(p) \neq \emptyset \text{ for all } p \in X$$

$*$ A dense set pins down every neighborhood assignment

Pinning down number

$U : X \to \tau_X$ is a neighborhood assignment on X iff

$$p \in U(p) \text{ for all } p \in X$$

$A \subset X$ pins down a neighborhood assignment U iff

$$A \cap U(p) \neq \emptyset \text{ for all } p \in X$$

$*$ A dense set pins down every neighborhood assignment

Definition (T. Banakh, A. Ravsky)

pinning down number of a space X:

$$\mathrm{pd}(X)=\min\{\kappa : \forall U \in NEA(X)\ \exists A \in [X]^{\kappa}\ (A \text{ pins down } U)\}$$

Pinning down number

$U : X \to \tau_X$ is a neighborhood assignment on X iff

$$p \in U(p) \text{ for all } p \in X$$

$A \subset X$ pins down a neighborhood assignment U iff

$$A \cap U(p) \neq \emptyset \text{ for all } p \in X$$

$*$ A dense set pins down every neighborhood assignment

Definition (T. Banakh, A. Ravsky)

pinning down number of a space X:

$$\mathrm{pd}(X) = \min\{\kappa : \forall U \in NEA(X)\ \exists A \in [X]^{\kappa}\ (A \text{ pins down } U)\}$$

$\star$ $\mathrm{pd}(X) \leq \mathrm{d}(X)$.

Pinning down number

$U : X \to \tau_X$ is a neighborhood assignment on X iff

$$p \in U(p) \text{ for all } p \in X$$

$A \subset X$ pins down a neighborhood assignment U iff

$$A \cap U(p) \neq \emptyset \text{ for all } p \in X$$

$*$ A dense set pins down every neighborhood assignment

Definition (T. Banakh, A. Ravsky)

pinning down number of a space X:

$$\mathrm{pd}(X)=\min\{\kappa : \forall U \in NEA(X)\ \exists A \in [X]^{\kappa}\ (A \text{ pins down } U)\}$$

$\star$ $\mathrm{pd}(X) \leq \mathrm{d}(X)$.

- T. Banakh, A. Ravsky: $e^{-}(X)$, foredensity ;
- Aurichi, Bella: $\mathrm{d}_{NA}(X)$,

First results

- U is a NEA on X iff $U : X \to \tau_X$ s.t. $a \in U(a)$ for all $a \in X$
- $P \subset X$ pins down a nea U iff $P \cap U(a) \neq \emptyset$ for all $a \in X$
- pd(X)= min$\{\kappa : \forall U \in NEA(X)\ \exists A \in \left[X\right]^{\kappa}\ (A \text{ pins down } U)\}$
- pd(X) $\leq$ d(X).

First results

- U is a NEA on X iff $U : X \to \tau_X$ s.t. $a \in U(a)$ for all $a \in X$
- $P \subset X$ pins down a nea U iff $P \cap U(a) \neq \emptyset$ for all $a \in X$
- pd(X)= $\min\{\kappa : \forall U \in NEA(X)\ \exists A \in [X]^{\kappa}\ (A$ pins down $U)\}$
- $\mathrm{pd}(X) \leq \mathrm{d}(X)$.

Theorem (T. Banakh, A. Ravsky)

- *If X is T_2, $|X| < \aleph_\omega$, then* $\mathrm{pd}(X) = \mathrm{d}(X)$.
- *If $2^{2^{\mathrm{cf}(\kappa)}} > \kappa > \mathrm{cf}(\kappa)$, then there is a T_2 space X with $pd(X) < \mathrm{d}(X)$.*

First results

- U is a NEA on X iff $U : X \to \tau_X$ s.t. $a \in U(a)$ for all $a \in X$
- $P \subset X$ pins down a nea U iff $P \cap U(a) \neq \emptyset$ for all $a \in X$
- $\mathrm{pd}(X) = \min\{\kappa : \forall U \in NEA(X)\ \exists A \in [X]^{\kappa}\ (A \text{ pins down } U)\}$
- $\mathrm{pd}(X) \leq \mathrm{d}(X)$.

Theorem (T. Banakh, A. Ravsky)

- *If X is T_2, $|X| < \aleph_\omega$, then* $\mathrm{pd}(X) = \mathrm{d}(X)$.
- *If $2^{2^{\mathrm{cf}(\kappa)}} > \kappa > \mathrm{cf}(\kappa)$, then there is a T_2 space X with $pd(X) < \mathrm{d}(X)$.*

A topological space X is a pd-example iff $pd(X) < d(X)$.

First results

- U is a NEA on X iff $U : X \to \tau_X$ s.t. $a \in U(a)$ for all $a \in X$
- $P \subset X$ pins down a nea U iff $P \cap U(a) \neq \emptyset$ for all $a \in X$
- pd(X)= $\min\{\kappa : \forall U \in NEA(X)\ \exists A \in \left[X\right]^{\kappa}\ (A \text{ pins down } U)\}$
- $\mathrm{pd}(X) \leq \mathrm{d}(X)$.

Theorem (T. Banakh, A. Ravsky)

- *If X is T_2, $|X| < \aleph_\omega$, then $\mathrm{pd}(X) = \mathrm{d}(X)$.*
- *If $2^{2^{\mathrm{cf}(\kappa)}} > \kappa > \mathrm{cf}(\kappa)$, then there is a T_2 space X with $pd(X) < \mathrm{d}(X)$.*

A topological space X is a pd-example iff $pd(X) < d(X)$.

Questions

- *Regular pd-example?*
- *ZFC pd-example?*

First equivalence

- U is a NEA on X iff $U : X \to \tau_X$ s.t. $a \in U(a)$ for all $a \in X$
- $P \subset X$ pins down a nea U iff $P \cap U(a) \neq \emptyset$ for all $a \in X$
- $\mathrm{pd}(X) = \min\{\kappa : \forall U \in NEA(X)\ \exists A \in \left[X\right]^{\kappa}\ (A \text{ pins down } U)\}$

First equivalence

- U is a NEA on X iff $U : X \to \tau_X$ s.t. $a \in U(a)$ for all $a \in X$
- $P \subset X$ pins down a nea U iff $P \cap U(a) \neq \emptyset$ for all $a \in X$
- $\mathrm{pd}(X) = \min\{\kappa : \forall U \in NEA(X)\ \exists A \in \left[X\right]^{\kappa}\ (A \text{ pins down } U)\}$

Theorem (I. Juhász, L.S., Z. Szentmiklóssy)

T.F.A.E:

(1) $2^{\kappa} < \kappa^{+\omega}$ *for each cardinal* κ,

(2) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each* T_2 *space* X,

(3) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each 0-dimensional* T_2 *space* X.

A special case

A special case

- dispersion character

$$\Delta(X) = \min\{|U| : \emptyset \neq U \subset^{open} X\}.$$

A special case

- dispersion character

$$\Delta(X) = \min\{|U| : \emptyset \neq U \subset^{open} X\}.$$

- X is neat: $|X| = \Delta(X)$

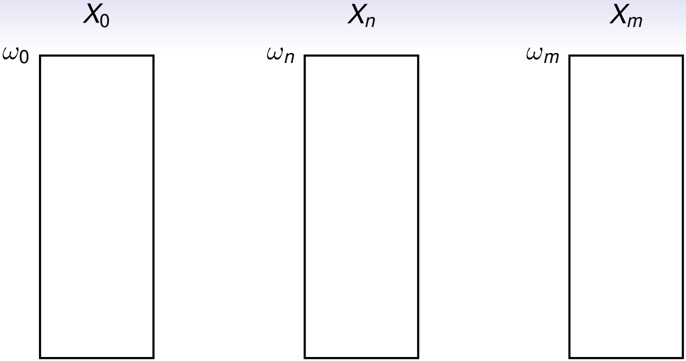
A special case

- dispersion character

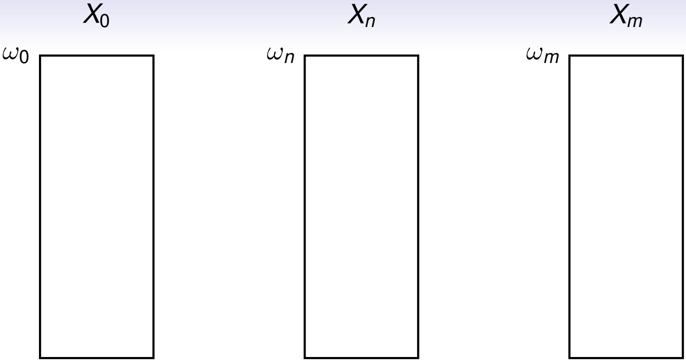
$$\Delta(X) = \min\{|U| : \emptyset \neq U \subset^{open} X\}.$$

- X is neat: $|X| = \Delta(X)$

We prove:
If $2^\omega > \omega_\omega$ then there is a 0-dimensional space X with $pd(X) = \omega$ and $|X| = \Delta(X) = d(X) = \omega_\omega$.

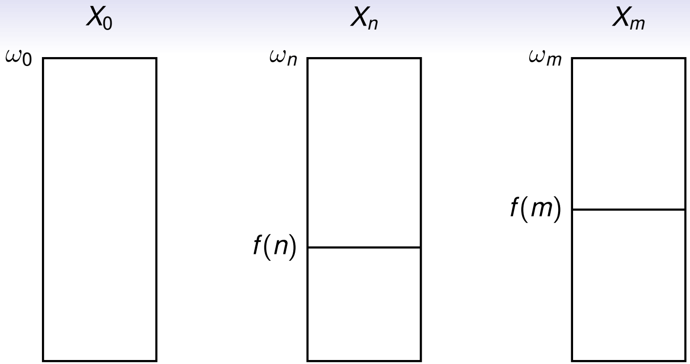


• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.



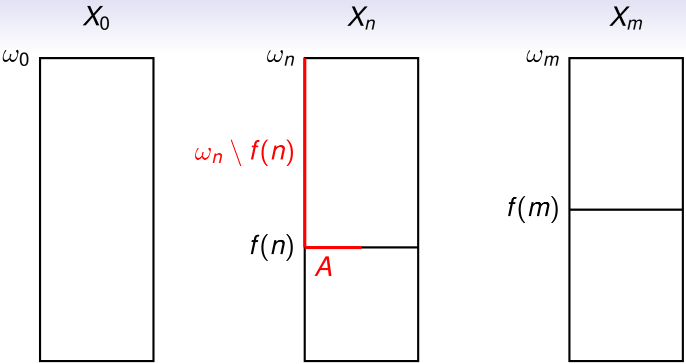
- $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \bigcup_{m \geq n} \Big((\omega_m \setminus f(m)) \times A \Big)$.



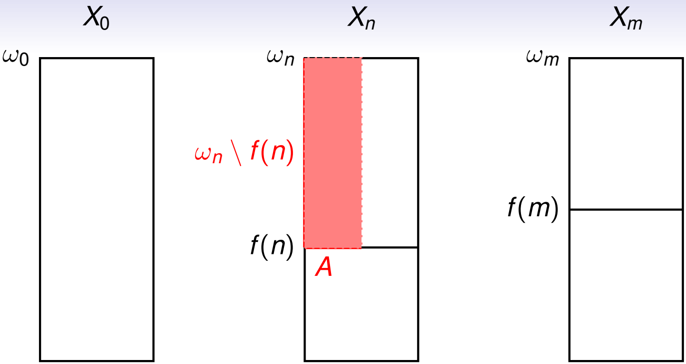
- $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \bigcup_{m \geq n} \Big((\omega_m \setminus f(m)) \times A \Big)$.



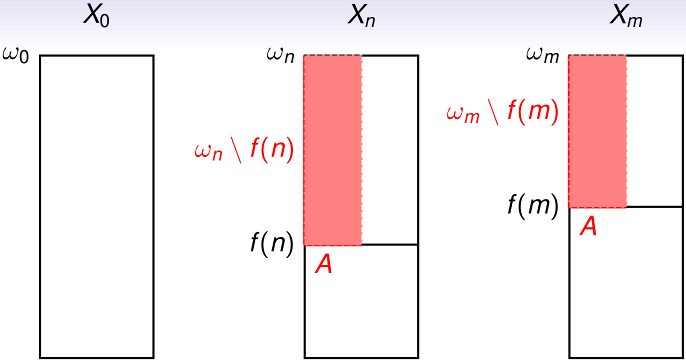
- $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \bigcup_{m \geq n} \Big((\omega_m \setminus f(m)) \times A \Big)$.



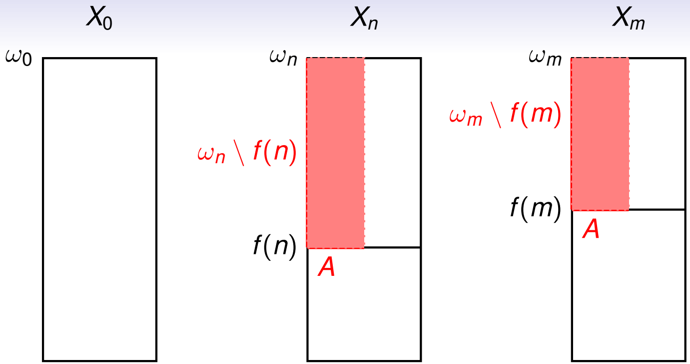
- $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \bigcup_{m \geq n} \Big((\omega_m \setminus f(m)) \times A \Big)$.



- $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \bigcup_{m \geq n} \Big((\omega_m \setminus f(m)) \times A \Big)$.

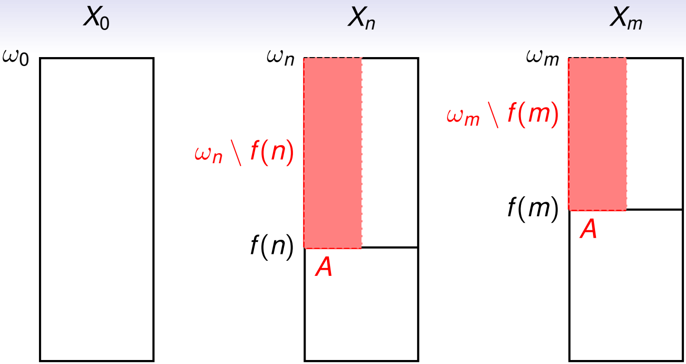


• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod (\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \bigcup_{m \geq n} \Big((\omega_m \setminus f(m)) \times A \Big)$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase of τ: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$



• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \bigcup_{m \geq n} \Big((\omega_m \setminus f(m)) \times A \Big)$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase of τ: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A \in \langle \mathcal{A} \rangle$

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $d(X) = \omega_\omega$.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.
If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \ge n} (\omega_m \setminus f(m)) \right) \times A$.
Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $d(X) = \omega_\omega$.

- Assume $|D| < \omega_\omega$.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $d(X) = \omega_\omega$.

- Assume $|D| < \omega_\omega$.
- $|D| < \omega_n$ for some n

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $d(X) = \omega_\omega$.

- Assume $|D| < \omega_\omega$.
- $|D| < \omega_n$ for some n
- there is $f \in \mathbb{P}$ such that $D \cap X_m \subset f(m) \times \omega$ for $m \geq n$.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $d(X) = \omega_\omega$.

- Assume $|D| < \omega_\omega$.
- $|D| < \omega_n$ for some n
- there is $f \in \mathbb{P}$ such that $D \cap X_m \subset f(m) \times \omega$ for $m \geq n$.
- Then $G(n, f, A_{n,f}) \cap D = \emptyset$.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $d(X) = \omega_\omega$.

- Assume $|D| < \omega_\omega$.
- $|D| < \omega_n$ for some n
- there is $f \in \mathbb{P}$ such that $D \cap X_m \subset f(m) \times \omega$ for $m \geq n$.
- Then $G(n, f, A_{n,f}) \cap D = \emptyset$.
- Thus D is not dense.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n,f,A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n,f,A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n,f,A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.
- $\exists n \geq n_p$ s.t. $f_p(n) < g(n)$

• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

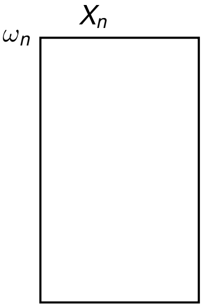
Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.
- $\exists n \geq n_p$ s.t. $f_p(n) < g(n)$



• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod (\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

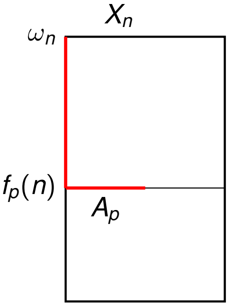
Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.
- $\exists n \geq n_p$ s.t. $f_p(n) < g(n)$



• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

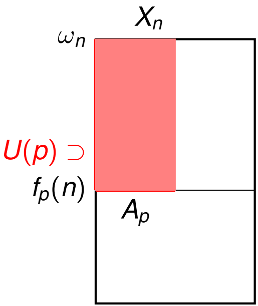
Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.
- $\exists n \geq n_p$ s.t. $f_p(n) < g(n)$



• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

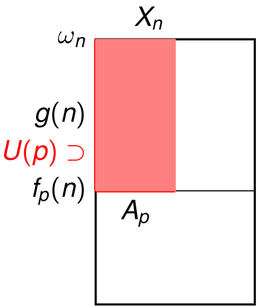
Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.
- $\exists n \geq n_p$ s.t. $f_p(n) < g(n)$



• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \Big(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \Big) \times A$.

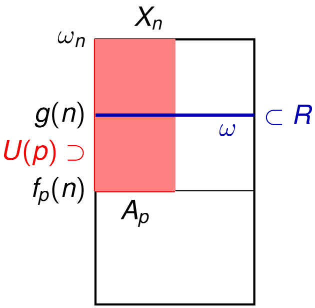
Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.
- $\exists n \geq n_p$ s.t. $f_p(n) < g(n)$



• $X = \langle \omega_\omega \times \omega, \tau \rangle$ • $X_n = (\omega_n \setminus \omega_{n-1}) \times \omega$. • $\mathbb{P} = \prod(\omega_n \setminus \omega_{n-1})$.

If $n \in \omega$, $f \in \mathbb{P}$, $A \subset \omega$ let $G(n, f, A) = \left(\bigcup_{m \geq n} (\omega_m \setminus f(m)) \right) \times A$.

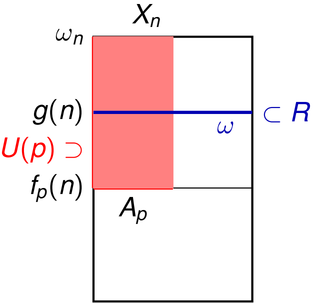
Fix an independent family $\mathcal{A} = \{A_{n,f} : n \in \omega, f \in \mathbb{P}\} \subset [\omega]^\omega$.

Clopen subbase: $\{G(n, f, A_{n,f}) : n \in \omega, f \in \mathbb{P}.\}$

If $\emptyset \neq U \subset^{open} X$ then $G(n, f, A) \subset U$ for some $n \in \omega$, $f \in \prod$, $A_U \in \langle \mathcal{A} \rangle$

Claim: $pd(X) = \omega$.

- $U : X \to \tau$ be a NEA.
- Then $U(p) \supset G(n_p, f_p, A_p)$ for all $p \in X$
- there is $g \in \mathbb{P}$ s.t. $f_p <^* g$ for all $p \in X$.
- $R = \{g(n) : n \in \omega\} \times \omega$ pins down U
- Let $p \in X$. Then $U(p) \supset G(n_p, f_p, A_p)$.
- $\exists n \geq n_p$ s.t. $f_p(n) < g(n)$
- Then $R \cap X_n \cap G(n_p, f_p, A_p) \neq \emptyset$.



Some observations

Some observations

If $\mathrm{pd}(X) < \mathrm{d}(X)$, then $\exists Y \subset^{open} X$ s.t. $\mathrm{pd}(Y) < \mathrm{d}(Y)$ and $\Delta(Y) = |Y|$.

Some observations

If $\mathrm{pd}(X) < \mathrm{d}(X)$, then $\exists Y \subset^{open} X$ s.t. $\mathrm{pd}(Y) < \mathrm{d}(Y)$ and $\Delta(Y) = |Y|$.

First pd-examples:

$$\mathrm{pd}(X) = \mathrm{cf}(|X|) < \mathrm{d}(X) = \Delta(X) = |X|.$$

Some observations

If $\mathrm{pd}(X) < \mathrm{d}(X)$, then $\exists Y \subset^{open} X$ s.t. $\mathrm{pd}(Y) < \mathrm{d}(Y)$ and $\Delta(Y) = |Y|$.

First pd-examples:

$$\mathrm{pd}(X) = \mathrm{cf}(|X|) < \mathrm{d}(X) = \Delta(X) = |X|.$$

Questions

- *Can* $\mathrm{d}(X)$ *be a regular cardinal?*
- *Can* $|X|$ *be a regular cardinal?*

Some observations

If $\mathrm{pd}(X) < \mathrm{d}(X)$, then $\exists Y \subset^{open} X$ s.t. $\mathrm{pd}(Y) < \mathrm{d}(Y)$ and $\Delta(Y) = |Y|$.

First pd-examples:

$$\mathrm{pd}(X) = \mathrm{cf}(|X|) < \mathrm{d}(X) = \Delta(X) = |X|.$$

Questions

- *Can* $\mathrm{d}(X)$ *be a regular cardinal?*
- *Can* $|X|$ *be a regular cardinal?*

Modified construction:

$$\mathrm{pd}(X) = \mathrm{cf}(|X|) < \mathrm{d}(X) = \mathrm{cf}(\mathrm{d}(X)) < \Delta(X) = |X|$$

Shelah's Strong Hypothesis

Shelah's Strong Hypothesis

- $\mu > \mathrm{cf}(\mu)$

Shelah's Strong Hypothesis

- $\mu > \mathrm{cf}(\mu)$
- $S(\mu) = \{a \in [\mu \cap \mathfrak{Reg}]^{\mathrm{cf}(\mu)} : \sup a = \mu\}$

Shelah's Strong Hypothesis

- $\mu > \text{cf}(\mu)$
- $S(\mu) = \{a \in [\mu \cap \mathfrak{Reg}]^{\text{cf}(\mu)} : \sup a = \mu\}$
- $\mathcal{U}(a) = \{D : D \text{ is an ultrafilter on } a, D \cap J^{bd}[a] = \emptyset\}$.

Shelah's Strong Hypothesis

- $\mu > \text{cf}(\mu)$
- $S(\mu) = \{a \in [\mu \cap \mathfrak{Reg}]^{\text{cf}(\mu)} : \sup a = \mu\}$
- $\mathcal{U}(a) = \{D : D \text{ is an ultrafilter on } a, D \cap J^{bd}[a] = \emptyset\}$.
- $\text{pp}(\mu) = \sup\{\text{cf}(\prod a/D) : a \in S(\mu), D \in \mathcal{U}(a)\}$

Shelah's Strong Hypothesis

- $\mu > \mathrm{cf}(\mu)$
- $S(\mu) = \{a \in [\mu \cap \mathfrak{Reg}]^{\mathrm{cf}(\mu)} : \sup a = \mu\}$
- $\mathcal{U}(a) = \{D : D \text{ is an ultrafilter on } a, D \cap J^{bd}[a] = \emptyset\}$.
- $\mathrm{pp}(\mu) = \sup\{\mathrm{cf}(\prod a/D) : a \in S(\mu), D \in \mathcal{U}(a))\}$

Shelah's Strong Hypothesis:

$$\mathrm{pp}(\mu) = \mu^+ \text{ for all singular cardinal } \mu.$$

An equiconsistency result

Theorem (I. Juhász, L.S., Z. Szentmiklóssy)

The following three statements are equiconsistent*:*

(i) *There is a singular cardinal* λ *with* $pp(\lambda) > \lambda^+$, *i.e. Shelah's Strong Hypothesis fails;*

(ii) *there is a 0-dimensional Hausdorff space* X *such that* $|X| = \Delta(X)$ *is a regular cardinal and* $pd(X) < d(X)$*;*

(iii) *there is a topological space* X *such that* $|X| = \Delta(X)$ *is a regular cardinal and* $pd(X) < d(X)$.

An equiconsistency result

Theorem (I. Juhász, L.S., Z. Szentmiklóssy)

The following three statements are equiconsistent*:*

(i) *There is a singular cardinal* λ *with* $pp(\lambda) > \lambda^+$, *i.e. Shelah's Strong Hypothesis fails;*

(ii) *there is a 0-dimensional Hausdorff space* X *such that* $|X| = \Delta(X)$ *is a regular cardinal and* $pd(X) < d(X)$*;*

(iii) *there is a topological space* X *such that* $|X| = \Delta(X)$ *is a regular cardinal and* $pd(X) < d(X)$*.*

No equivalence:
Con(failure of SSH + the limit cardinals are strong limit)

Connected and locally connected spaces

Theorem (I. Juhász, J. van Mill, L.S., Z. Szentmiklóssy)

T:F.A.E:

(1) $2^\kappa < \kappa^{+\omega}$ *for each cardinal* κ*,*

(2) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each* T_2 *space* X*,*

(3) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each 0-dimensional* T_2 *space* X*.*

Theorem (I. Juhász, J. van Mill, L.S., Z. Szentmiklóssy)

T:F.A.E:

(1) $2^\kappa < \kappa^{+\omega}$ *for each cardinal* κ,

(2) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each* T_2 *space* X,

(3) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each 0-dimensional* T_2 *space* X.

(4) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all connected, locally connected regular spaces.*

(5) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all Abelian topological groups.*

Theorem (I. Juhász, J. van Mill, L.S., Z. Szentmiklóssy)

T:F.A.E:

(1) $2^\kappa < \kappa^{+\omega}$ *for each cardinal* κ*,*

(2) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each* T_2 *space* X*,*

(3) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each 0-dimensional* T_2 *space* X*.*

(4) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all connected, locally connected regular spaces.*

(5) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all Abelian topological groups.*

What about connected Tychonoff spaces?

Theorem (I. Juhász,J. van Mill, L.S., Z. Szentmiklóssy)

T:F.A.E:

(1) $2^\kappa < \kappa^{+\omega}$ *for each cardinal* κ*,*

(2) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each* T_2 *space* X*,*

(3) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each 0-dimensional* T_2 *space* X*.*

(4) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all connected, locally connected regular spaces.*

(5) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all Abelian topological groups.*

What about connected Tychonoff spaces?

Theorem (JvMSSz)

It is consistent that

Connected and locally connected spaces

Theorem (I. Juhász, J. van Mill, L.S., Z. Szentmiklóssy)

T:F.A.E:

(1) $2^\kappa < \kappa^{+\omega}$ *for each cardinal* κ,

(2) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each* T_2 *space* X,

(3) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each 0-dimensional* T_2 *space* X.

(4) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all connected, locally connected regular spaces.*

(5) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all Abelian topological groups.*

What about connected Tychonoff spaces?

Theorem (JvMSSz)

It is consistent that

- *there is a 0-dimensional space* X *with* $\mathrm{pd}(X) < \mathrm{d}(X)$

Theorem (I. Juhász, J. van Mill, L.S., Z. Szentmiklóssy)
T:F.A.E:

(1) $2^\kappa < \kappa^{+\omega}$ *for each cardinal* κ,

(2) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each* T_2 *space* X,

(3) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for each 0-dimensional* T_2 *space* X.

(4) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all connected, locally connected regular spaces.*

(5) $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all Abelian topological groups.*

What about connected Tychonoff spaces?

Theorem (JvMSSz)
It is consistent that

- *there is a 0-dimensional space* X *with* $\mathrm{pd}(X) < \mathrm{d}(X)$
- $\mathrm{pd}(X) = \mathrm{d}(X)$ *for all connected Tychonoff spaces.*

A connected, locally connected Tychonoff pd-example

A connected, locally connected Tychonoff pd-example

If X is a connected, Tychonoff space then $|X| \geq 2^{\omega}$.

A connected, locally connected Tychonoff pd-example

If X is a connected, Tychonoff space then $|X| \geq 2^\omega$.

Theorem (I. Juhász,J. van Mill, L.S., Z. Szentmiklóssy)
T:F.A.E:

(1) *There is a singular cardinal $\mu \geq 2^\omega$ which is not a strong limit cardinal.*

(2) *There is a neat, connected, locally connected Tychonoff space X with singular $\Delta(X) = |X|$ and $\mathrm{pd}(X) < \mathrm{d}(X)$.*

If X is a connected, Tychonoff space then $|X| \geq 2^\omega$.

Theorem (I. Juhász,J. van Mill, L.S., Z. Szentmiklóssy)
T:F.A.E:

(1) *There is a singular cardinal $\mu \geq 2^\omega$ which is not a strong limit cardinal.*

(2) *There is a neat, connected, locally connected Tychonoff space X with singular $\Delta(X) = |X|$ and $\mathrm{pd}(X) < \mathrm{d}(X)$.*

(3) *There is a neat, pathwise connected, locally pathwise connected Tychonoff Abelian topological group X with singular $\Delta(X) = |X|$ and $\mathrm{pd}(X) < \mathrm{d}(X)$.*

Extension theorems

Extension theorems

connected T_3 pd-example

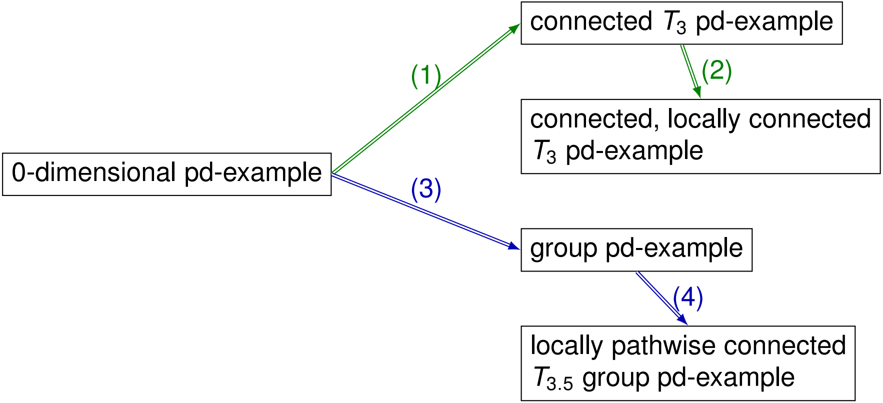
connected, locally connected T_3 pd-example

0-dimensional pd-example

group pd-example

locally pathwise connected $T_{3.5}$ group pd-example

Extension theorems



T_3 pd-example$\Longrightarrow$ connected T_3 pd-example

T_3 pd-example$\Longrightarrow$ connected T_3 pd-example

- Assume that X is a T_3 pd-example.

T_3 pd-example$\Longrightarrow$ connected T_3 pd-example

- Assume that X is a T_3 pd-example.
- Ciesielski and Wojciechowsk: there is a separable connected T_3 space P of size ω_1

T_3 pd-example$\Longrightarrow$ connected T_3 pd-example

- Assume that X is a T_3 pd-example.
- Ciesielski and Wojciechowsk: there is a separable connected T_3 space P of size ω_1
- Fix $p \in P$. The underlying set of Z is

$$\big(X \times (P \setminus \{p\})\big) \cup \{\infty\}.$$

T_3 pd-example $\Longrightarrow$ connected T_3 pd-example

- Assume that X is a T_3 pd-example.
- Ciesielski and Wojciechowsk: there is a separable connected T_3 space P of size ω_1
- Fix $p \in P$. The underlying set of Z is

$$\big(X \times (P \setminus \{p\})\big) \cup \{\infty\}.$$

- Topology on $X \times (P \setminus \{p\})$ in Z is the product topology.
  A basic neighborhood of ∞ has the form

$$\big(X \times (U \setminus \{p\})\big) \cup \{\infty\},$$

where U is any neighborhood of p in P.

T_3 pd-example$\Longrightarrow$ connected T_3 pd-example

- Assume that X is a T_3 pd-example.
- Ciesielski and Wojciechowsk: there is a separable connected T_3 space P of size ω_1
- Fix $p \in P$. The underlying set of Z is

$$\big(X \times (P \setminus \{p\})\big) \cup \{\infty\}.$$

- Topology on $X \times (P \setminus \{p\})$ in Z is the product topology.

  A basic neighborhood of ∞ has the form

$$\big(X \times (U \setminus \{p\})\big) \cup \{\infty\},$$

  where U is any neighborhood of p in P.
- Theorem: Z is connected T_3, $d(X) = d(Z)$ and $pd(X) = pd(Z)$.

connected T_3 pd-example $\Longrightarrow$ connected, loc. connected T_3 pd-example

connected T_3 pd-example$\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX

connected T_3 pd-example $\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX
- $\mathcal{L}$ is linked system if any two of its members meet.

connected T_3 pd-example$\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX
- $\mathcal{L}$ is linked system if any two of its members meet.
- $$\lambda X = \{\mathcal{L} : \mathcal{L} \text{ is a maximal linked family of of closed subsets of } X. \}$$

connected T_3 pd-example $\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX
- $\mathcal{L}$ is linked system if any two of its members meet.
- $$\lambda X = \{\mathcal{L} : \mathcal{L} \text{ is a maximal linked family of of closed subsets of } X.\}$$
- For $A \subset X$ let
$$A^+ = \{\mathcal{M} \in \lambda X : (\exists\, M \in \mathcal{M})(M \subset A)\}.$$

connected T_3 pd-example $\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX
- $\mathcal{L}$ is linked system if any two of its members meet.
- $$\lambda X = \{\mathcal{L} : \mathcal{L} \text{ is a maximal linked family of of closed subsets of } X.\}$$
- For $A \subset X$ let
$$A^+ = \{\mathcal{M} \in \lambda X : (\exists\, M \in \mathcal{M})(M \subset A)\}.$$
- closed subbase of λX:
$$\{A^+ : A \text{ is closed in } X\}$$

connected T_3 pd-example $\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX
- $\mathcal{L}$ is linked system if any two of its members meet.
- $$\lambda X = \{\mathcal{L} : \mathcal{L} \text{ is a maximal linked family of of closed subsets of } X.\}$$
- For $A \subset X$ let
$$A^+ = \{\mathcal{M} \in \lambda X : (\exists M \in \mathcal{M})(M \subset A)\}.$$
- closed subbase of λX:
$$\{A^+ : A \text{ is closed in } X\}$$
- $\lambda_f X = \{\mathcal{L} \in \lambda X : \exists M \in [X]^{<\omega}\ (\forall L \in \mathcal{L})\ L \cap M \in \mathcal{L}\}$

connected T_3 pd-example $\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX
- $\mathcal{L}$ is linked system if any two of its members meet.
- $$\lambda X = \{\mathcal{L} : \mathcal{L} \text{ is a maximal linked family of of closed subsets of } X.\ \}$$
- For $A \subset X$ let
$$A^+ = \{\mathcal{M} \in \lambda X : (\exists\, M \in \mathcal{M})(M \subset A)\}.$$
- closed subbase of λX:
$$\{A^+ : A \text{ is closed in } X\}$$
- $\lambda_f X = \{\mathcal{L} \in \lambda X : \exists M \in [X]^{<\omega}\ (\forall L \in \mathcal{L})\ L \cap M \in \mathcal{L}\}$
- Verbeek: X is connected $\Longrightarrow \lambda_f X$ is connected and locally connected

connected T_3 pd-example $\Longrightarrow$ connected, loc. connected T_3 pd-example

- de Groot introduced the superextension of X denoted by λX
- $\mathcal{L}$ is linked system if any two of its members meet.
- $$\lambda X = \{\mathcal{L} : \mathcal{L} \text{ is a maximal linked family of of closed subsets of } X.\ \}$$
- For $A \subset X$ let
$$A^+ = \{\mathcal{M} \in \lambda X : (\exists\, M \in \mathcal{M})(M \subset A)\}.$$
- closed subbase of λX:
$$\{A^+ : A \text{ is closed in } X\}$$
- $\lambda_f X = \{\mathcal{L} \in \lambda X : \exists M \in [X]^{<\omega}\ (\forall L \in \mathcal{L})\ L \cap M \in \mathcal{L}\}$
- Verbeek: X is connected $\Longrightarrow \lambda_f X$ is connected and locally connected
- JvMSSz: $d(X) = d(\lambda_f X)$ and $pd(X) = pd(\lambda_f X)$

pd-example $\Longrightarrow$ (Abelian) group pd-example

pd-example$\Longrightarrow$ (Abelian) group pd-example

If X is a $T_{3.5}$-space, then $F(X)$ and $A(X)$ denote the free topological group and the free abelian topological group on X.

pd-example$\Longrightarrow$ (Abelian) group pd-example

If X is a $T_{3.5}$-space, then $F(X)$ and $A(X)$ denote the free topological group and the free abelian topological group on X.

$F(X)$ is a topological group containing (a homeomorphic copy of) X such that

1. X generates $F(X)$ algebraically,
2. every continuous function $f : X \to H$, where H is any topological group, can be extended to a continuous homomorphism $\overline{f} : F(X) \to H$.

pd-example $\Longrightarrow$ (Abelian) group pd-example

If X is a $T_{3.5}$-space, then $F(X)$ and $A(X)$ denote the free topological group and the free abelian topological group on X.

$F(X)$ is a topological group containing (a homeomorphic copy of) X such that

1. X generates $F(X)$ algebraically,
2. every continuous function $f : X \to H$, where H is any topological group, can be extended to a continuous homomorphism $\overline{f} : F(X) \to H$.

Similarly for $A(X)$.

pd-example $\Longrightarrow$ (Abelian) group pd-example

If X is a $T_{3.5}$-space, then $F(X)$ and $A(X)$ denote the free topological group and the free abelian topological group on X.
$F(X)$ is a topological group containing (a homeomorphic copy of) X such that

1. X generates $F(X)$ algebraically,
2. every continuous function $f : X \to H$, where H is any topological group, can be extended to a continuous homomorphism $\overline{f} : F(X) \to H$.

Similarly for $A(X)$.
The existence of these groups was proved by Markov.

pd-example $\Longrightarrow$ (Abelian) group pd-example

If X is a $T_{3.5}$-space, then $F(X)$ and $A(X)$ denote the free topological group and the free abelian topological group on X.
$F(X)$ is a topological group containing (a homeomorphic copy of) X such that

1. X generates $F(X)$ algebraically,
2. every continuous function $f : X \to H$, where H is any topological group, can be extended to a continuous homomorphism $\overline{f} : F(X) \to H$.

Similarly for $A(X)$.
The existence of these groups was proved by Markov.

Theorem (JvMSSz)

Let X be a $T_{3.5}$-space. Then

$$d(X) = d(F(X)) = d(A(X)).$$

If X is neat, then so are $A(X)$ and $F(X)$, and

$$pd(X) = pd(A(X)) = pd(F(X)).$$

group pd-example $\Longrightarrow$ (loc) pathwise-connected group pd-example

group pd-example$\Longrightarrow$ (loc) pathwise-connected group pd-example

- Hartman Mycielski construction

group pd-example $\Longrightarrow$ (loc) pathwise-connected group pd-example

- Hartman Mycielski construction
- Let $(G, \cdot, e)$ be a Tychonoff topological group.

$$G^\bullet = \big\{ f \in {}^{[0,1)}G : \text{ for some sequence } 0 = a_0 < a_1 < \cdots < a_n = 1 \; f \text{ is constant on } [a_k, a_{k+1}) \text{ for every } k = 0, \ldots, n{-}1 \big\}.$$

group pd-example$\Longrightarrow$ (loc) pathwise-connected group pd-example

- Hartman Mycielski construction
- Let $(G, \cdot, e)$ be a Tychonoff topological group.

$$G^\bullet = \{ f \in {}^{[0,1)}G : \text{ for some sequence } 0 = a_0 < a_1 < \cdots < a_n = 1 \; f \text{ is constant on } [a_k, a_{k+1}) \text{ for every } k = 0, \ldots, n-1 \}.$$

- Define $*$ on $G^\bullet$ by $(f * g)(x) = f(x) \cdot g(x)$ for all $f, g \in G^\bullet$ and $x \in [0, 1)$.

group pd-example $\Longrightarrow$ (loc) pathwise-connected group pd-example

- Hartman Mycielski construction
- Let $(G, \cdot, e)$ be a Tychonoff topological group.

$$G^\bullet = \big\{ f \in {}^{[0,1)}G : \text{for some sequence } 0 = a_0 < a_1 < \cdots < a_n = 1 \; f \text{ is constant on } [a_k, a_{k+1}) \text{ for every } k = 0, \ldots, n{-}1 \big\}.$$

- Define $*$ on $G^\bullet$ by $(f * g)(x) = f(x) \cdot g(x)$ for all $f, g \in G^\bullet$ and $x \in [0, 1)$.
- $(G^\bullet, *, e^\bullet)$ is a group, where $e^\bullet(r) = e$ for each $r \in [0, 1)$.

group pd-example $\Longrightarrow$ (loc) pathwise-connected group pd-example

- Hartman Mycielski construction
- Let $(G, \cdot, e)$ be a Tychonoff topological group.

$$G^\bullet = \{f \in {}^{[0,1)}G : \text{for some sequence } 0 = a_0 < a_1 < \cdots < a_n = 1 \; f \text{ is constant on } [a_k, a_{k+1}) \text{ for every } k = 0, \ldots, n-1\}.$$

- Define $*$ on $G^\bullet$ by $(f * g)(x) = f(x) \cdot g(x)$ for all $f, g \in G^\bullet$ and $x \in [0, 1)$.
- $(G^\bullet, *, e^\bullet)$ is a group, where $e^\bullet(r) = e$ for each $r \in [0, 1)$.
- G embeds into $G^\bullet$ via $x \mapsto x^\bullet$, where $x^\bullet(r) = x$ for every $r \in [0, 1)$.

group pd-example$\Longrightarrow$ (loc) pathwise-connected group pd-example

- Hartman Mycielski construction
- Let $(G, \cdot, e)$ be a Tychonoff topological group.

$$G^\bullet = \big\{ f \in {}^{[0,1)}G : \text{for some sequence } 0 = a_0 < a_1 < \cdots < a_n = 1 \; f \text{ is constant on } [a_k, a_{k+1}) \text{ for every } k = 0, \ldots, n{-}1 \big\}.$$

- Define $*$ on $G^\bullet$ by $(f * g)(x) = f(x) \cdot g(x)$ for all $f, g \in G^\bullet$ and $x \in [0, 1)$.
- $(G^\bullet, *, e^\bullet)$ is a group, where $e^\bullet(r) = e$ for each $r \in [0, 1)$.
- G embeds into $G^\bullet$ via $x \mapsto x^\bullet$, where $x^\bullet(r) = x$ for every $r \in [0, 1)$.
- For $e \in V \in \tau_G$ and $\varepsilon > 0$, put

$$O(V, \varepsilon) = \{ f \in G^\bullet : \lambda(\{ r \in [0, 1) : f(r) \notin V \}) < \varepsilon \}$$

group pd-example$\Longrightarrow$ (loc) pathwise-connected group pd-example

- Hartman Mycielski construction
- Let $(G, \cdot, e)$ be a Tychonoff topological group.

$$G^\bullet = \big\{ f \in {}^{[0,1)}G : \text{ for some sequence } 0 = a_0 < a_1 < \cdots < a_n = 1 \quad f \text{ is constant on } [a_k, a_{k+1}) \text{ for every } k = 0, \ldots, n{-}1 \big\}.$$

- Define $*$ on $G^\bullet$ by $(f * g)(x) = f(x) \cdot g(x)$ for all $f, g \in G^\bullet$ and $x \in [0, 1)$.
- $(G^\bullet, *, e^\bullet)$ is a group, where $e^\bullet(r) = e$ for each $r \in [0, 1)$.
- G embeds into $G^\bullet$ via $x \mapsto x^\bullet$, where $x^\bullet(r) = x$ for every $r \in [0, 1)$.
- For $e \in V \in \tau_G$ and $\varepsilon > 0$, put

$$O(V, \varepsilon) = \{f \in G^\bullet : \lambda(\{r \in [0, 1) : f(r) \notin V\})\} < \varepsilon\}$$

- The $O(V, \varepsilon)$ are the neighborhoods of the element $e^\bullet$ of $G^\bullet$ that generate the topology.

Properties of Hartman Mycielski extension $G^{\bullet}$

Properties of Hartman Mycielski extension $G^\bullet$

Theorem

$G^\bullet$ is a topological group and is pathwise connected and locally pathwise connected.

$d(G^\bullet) \leq d(G)$.

Properties of Hartman Mycielski extension $G^\bullet$

Theorem

$G^\bullet$ is a topological group and is pathwise connected and locally pathwise connected.
$d(G^\bullet) \leq d(G)$.

Theorem (JvMSSz)

- $d(G) = d(G^\bullet)$.
- *If G is neat, and $|G| \geq 2^\omega$, then $G^\bullet$ is neat and $pd(G^\bullet) = pd(G)$.*

Positive results

Positive results

Theorem

If X is compact then $pd(X) = \mathrm{d}(X)$.

Positive results

Theorem
If X is compact then $pd(X) = \mathrm{d}(X)$.

Question (JSSz)

- *What about (regular) Lindelöf spaces?*
- *What about (regular) countably compact spaces?*

Positive results

Theorem

If X is compact then $pd(X) = \mathrm{d}(X)$.

Question (JSSz)

- *What about (regular) Lindelöf spaces?*
- *What about (regular) countably compact spaces?*

Theorem (Juhász,van Mill, S, Szentmiklóssy)

It is consistent that $\mathrm{pd}(X) < \mathrm{d}(X)$ for some hereditarily Lindelöf regular space X.

Positive results

Theorem

If X is compact then $pd(X) = \mathrm{d}(X)$.

Question (JSSz)

- *What about (regular) Lindelöf spaces?*
- *What about (regular) countably compact spaces?*

Theorem (Juhász,van Mill, S, Szentmiklóssy)

It is consistent that $\mathrm{pd}(X) < \mathrm{d}(X)$ for some hereditarily Lindelöf regular space X.

Theorem (Juhász, Shelah)

For any singular cardinal μ it is consistent that there is a hereditarily Lindelöf regular space X such that $d(X) = \mu$.

Positive results

Theorem

If X is compact then $pd(X) = \mathrm{d}(X)$.

Question (JSSz)

- *What about (regular) Lindelöf spaces?*
- *What about (regular) countably compact spaces?*

Theorem (Juhász,van Mill, S, Szentmiklóssy)

It is consistent that $\mathrm{pd}(X) < \mathrm{d}(X)$ for some hereditarily Lindelöf regular space X.

Theorem (Juhász, Shelah)

For any singular cardinal μ it is consistent that there is a hereditarily Lindelöf regular space X such that $d(X) = \mu$.

Fact: $pd(X) = cf(\mu)$.

Positive results

Theorem

If X is compact then $pd(X) = \mathrm{d}(X)$.

Question (JSSz)

- *What about (regular) Lindelöf spaces?*
- *What about (regular) countably compact spaces?*

Theorem (Juhász,van Mill, S, Szentmiklóssy)

It is consistent that $\mathrm{pd}(X) < \mathrm{d}(X)$ for some hereditarily Lindelöf regular space X.

Theorem (Juhász, Shelah)

For any singular cardinal μ it is consistent that there is a hereditarily Lindelöf regular space X such that $d(X) = \mu$.

Fact: $pd(X) = cf(\mu)$.

Problem

Is it consistent that there is a hereditarily Lindelöf regular space X such that $d(X) = 2^{\omega} > cf(2^{\omega})$?

Estimate d(X) using pd(X)

Estimate $\mathrm{d}(X)$ using $\mathrm{pd}(X)$

Theorem (JSSz)
$\mathrm{d}(X) \leq 2^{\mathrm{pd}(X)}$.

Estimate d(X) using pd(X)

Theorem (JSSz)
$d(X) \leq 2^{pd(X)}$.

Sharp?

Estimate d(X) using pd(X)

Theorem (JSSz)
$\mathrm{d}(X) \leq 2^{\mathrm{pd}(X)}$.

Sharp?

Theorem (JSSz)
$\mathrm{d}(X) < 2^{\mathrm{pd}(X)}$.

Estimate d(X) using pd(X)

Theorem (JSSz)
$\mathrm{d}(X) \leq 2^{\mathrm{pd}(X)}$.

Sharp?

Theorem (JSSz)
$\mathrm{d}(X) < 2^{\mathrm{pd}(X)}$.
Sharp?

Estimate $\mathrm{d}(X)$ using $\mathrm{pd}(X)$

Theorem (JSSz)
$\mathrm{d}(X) \leq 2^{\mathrm{pd}(X)}$.

Sharp?

Theorem (JSSz)
$\mathrm{d}(X) < 2^{\mathrm{pd}(X)}$.

Sharp?

Yes.
It is consistent that $2^{\mathrm{pd}(X)}$ is as large as you wish and $\mathrm{d}(X)^{+} = 2^{\mathrm{pd}(X)}$.

Inequalities

- Pospisil: $|X| \leq 2^{2^{\mathrm{d}(X)}}$ for T_2 spaces
- $\mathrm{w}(x) \leq 2^{\mathrm{d}(x)}$ for T_3 spaces

Inequalities

- Pospisil: $|X| \leq 2^{2^{\mathrm{d}(X)}}$ for T_2 spaces
- $\mathrm{w}(x) \leq 2^{\mathrm{d}(x)}$ for T_3 spaces

Theorem (JSSz)

$|X| \leq 2^{2^{\mathrm{pd}(X)}}$ *for* T_2 *spaces.*

Inequalities

- Pospisil: $|X| \leq 2^{2^{d(X)}}$ for T_2 spaces
- $w(x) \leq 2^{d(x)}$ for T_3 spaces

Theorem (JSSz)
$|X| \leq 2^{2^{pd(X)}}$ *for* T_2 *spaces.*

Theorem (JSSz)
If $|X| = \Delta(X)$, *then*

Inequalities

- Pospisil: $|X| \leq 2^{2^{\mathrm{d}(X)}}$ for T_2 spaces
- $\mathrm{w}(x) \leq 2^{\mathrm{d}(x)}$ for T_3 spaces

Theorem (JSSz)

$|X| \leq 2^{2^{\mathrm{pd}(X)}}$ *for* T_2 *spaces.*

Theorem (JSSz)

If $|X| = \Delta(X)$*, then*

- *either* $pd(X) = d(X)$ *and* $|X| \leq 2^{2^{\mathrm{pd}(X)}}$*, or*

Inequalities

- Pospisil: $|X| \leq 2^{2^{d(X)}}$ for T_2 spaces
- $w(x) \leq 2^{d(x)}$ for T_3 spaces

Theorem (JSSz)
$|X| \leq 2^{2^{pd(X)}}$ for T_2 spaces.

Theorem (JSSz)
If $|X| = \Delta(X)$, then

- *either $pd(X) = d(X)$ and $|X| \leq 2^{2^{pd(X)}}$, or*
- *$pd(X) < d(X)$ and $|X| < 2^{pd(X)}$.*

- Pospisil: $|X| \leq 2^{2^{\mathrm{d}(X)}}$ for T_2 spaces
- $\mathrm{w}(x) \leq 2^{\mathrm{d}(x)}$ for T_3 spaces

Theorem (JSSz)

$|X| \leq 2^{2^{\mathrm{pd}(X)}}$ for T_2 spaces.

Theorem (JSSz)

If $|X| = \Delta(X)$, then

- *either $pd(X) = d(X)$ and $|X| \leq 2^{2^{\mathrm{pd}(X)}}$, or*
- *$pd(X) < d(X)$ and $|X| < 2^{\mathrm{pd}(X)}$.*

Problem

Does $\mathrm{w}(x) \leq 2^{\mathrm{pd}(x)}$ hold for regular spaces?

The

6th European Set Theory Conference 2017

will be organized in **Budapest** from

July 3 – 7, 2017.

Homepage:

http://www.renyi.hu/∼6thestc