On κ-metrizable spaces

Andrzej Kucharski, Sławomir Turek

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The concept of a κ-metrizable spaces was introduced by Shchepin 1976.

All topological spaces under consideration are assumed to be at least Tichonov.

A set $A \subseteq X$ is *regular closed* if $\operatorname{cl} \operatorname{int} A = A$. For a topological space X let $\mathrm{RC}(X)$ denote the set of all regular closed sets and $\mathrm{CO}(X)$ denote the set of all closed and open sets .

Let (X,d) be a metrizable space. The distance

$$\rho(x,F)=\inf\{d(x,y):y\in F\}$$

has the following properties for any $F\in \mathrm{RC}(X)$:

(K1) $\rho(x,C)=0$ if and only if $x\in C$ for any $x\in X$ and $C\in \mathrm{RC}(X)$,

(K2) If $C\subseteq D$, then $\rho(x,C)\geq \rho(x,D)$ for any $x\in X$ and $C,D\in \mathrm{RC}(X)$,

(K3) $\rho(\cdot,C)$ is a continuous function for any $x\in X$,

(K4) $\rho(x,\mathrm{cl}(\bigcup_{\alpha<\lambda}C_\alpha))=\inf_{\alpha<\lambda}\rho(x,C_\alpha)$ for any non-decreasing totally ordered sequence $\{C_\alpha:\alpha<\lambda\}\subseteq \mathrm{RC}(X)$ and any $x\in X$.

A topological space X is *κ-metrizable* if there exists a non-negative function $\rho : X \times \mathrm{RC}(X) \to [0, \infty)$ satisfying the following axioms

(K1) $\rho(x, C) = 0$ if and only if $x \in C$ for any $x \in X$ and $C \in \mathrm{RC}(X)$,

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We say that ρ is *κ-metric* if it satisfies condition $(K1) - (K4)$.

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An example of κ-metrizable spaces

- all metrizable spaces are κ-metrizable,
- Sorgenfrey line with function $\rho(x, C) = d(x, C \cap [x, \infty))$ is κ-metrizable space,
- a dense (open, regular closed) subspace of a κ-metrizable space is κ-metrizable,
- Dugundji spaces (AE(0)),
- the product of any family of κ-metrizable spaces is κ-metrizable,
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and

$$(K4_\omega)\ \rho(x, \mathrm{cl}(\bigcup_{n<\omega} C_n)) = \inf_{n<\omega} \rho(x, C_n)$$

for any chain $\{C_n : n < \omega\} \subseteq \mathrm{RC}(X)$ and any $x \in X$,

then we say that ρ is *countable κ-metric.*

We say that X is countable *κ-metrizable* if there exists countable κ-metric.

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Note that if X satisfies countable chain condition then every countable κ-metric is κ-metric.

Let $\{C_\alpha : \alpha < \lambda\} \subseteq \mathrm{RC}(X)$ be a non-decreasing totally ordered sequence and $\lambda > \aleph_0$.

By countable chain condition there exists $\alpha < \lambda$ such that $C_\beta = C_\alpha$ for all $\alpha \leq \beta$.

Hence we get

$$\rho(x, \mathrm{cl}(\bigcup_{\alpha<\lambda} C_\alpha)) = \rho(x, C_{\alpha+1}) = \inf_{\alpha<\lambda} \rho(x, C_\alpha).$$

If $\lambda = \aleph_0$ then we apply condition $(K4_\omega)$.

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We give an example of countable κ-metric which is not κ-metric.

Example

Let τ be a measurable cardinal, i.e. an uncountable cardinal such that there exists a τ-complete nonprincipal ultrafilter ξ on τ. We consider $X = \tau \cup \{\xi\}$ with a topology inherit from Čech-Stone compactification of τ.

If τ is measurable cardinal (there exists $\aleph_1$-complete ultrafilter on τ) then there exists countable κ-metric two-valued.

If τ is the least cardinal that carries a countable κ-metric two-valued, then τ is measurable cardinal.

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Define a countable κ-metric by the followig condition

$$\rho(x, C) = \begin{cases} 1 & \text{if } x \notin C, \\ 0 & \text{if } x \in C \end{cases}$$

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Let $\{C_n : n \in \omega\} \subseteq \mathrm{RC}(X)$ be an increasing sequence and $x \in X$.

If there exists $n_0 \in \omega$ such that $x \in C_{n_0}$ then we get

$$\rho(x, \mathrm{cl} \bigcup_{n \in \omega} C_n) = 0 = \rho(x, C_{n_0}) = \inf\{\rho(x, C_n) : n \in \omega\}.$$

Otherwise $x \notin C_n$ for every $n \in \omega$.

Assume that $x \in \mathrm{cl} \bigcup_{n \in \omega} C_n$.

If $x \neq \xi$ then $\{x\} \cap \bigcup_{n \in \omega} C_n = \emptyset$, a contradiction.

If $x = \xi$ then $\tau \setminus C_n \in \xi$ for every $n \in \omega$. By $\aleph_1$-completnes $D = \bigcap_{n \in \omega} (\tau \setminus C_n) \in \xi$. Hence

$$(D \cup \{\xi\}) \cap \bigcup_{n \in \omega} C_n = \emptyset,$$

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But ρ is not κ-metric.

Let $C_\alpha = \alpha$ for $\alpha < \tau$. Then we have $\rho(\xi, \mathrm{cl} \bigcup \{C_\alpha : \alpha < \tau\}) = 0$ but $\inf\{\rho(\xi, C_\alpha) : \alpha < \tau\} = 1$.

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Assume that $\rho : X \times \mathrm{RC}(X) \to \{0,1\}$ is a countable κ-metric two-valued. We shall prove that ξ is $\aleph_1$-complete on τ.

Let $\{D_n : n \in \omega\} \subset \xi$. Suppose that $\bigcap\{D_n : n \in \omega\} \notin \xi$.

Then $\tau \setminus \bigcap\{D_n : n \in \omega\} = \bigcup\{F_n : n \in \omega\} \in \xi$, where $\tau \setminus D_1 \cup \ldots \cup \tau \setminus D_n = F_n \in \mathrm{CO}(X)$.

But $0 = \rho(\xi, \mathrm{cl} \bigcup\{F_n : n \in \omega\}) = \inf\{\rho(x, F_n) : n \in \omega\} = 1$, a contradiction.

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Question

Is there exist a countable κ-metrizable space which is not κ-metrizable in ZFC?

A continuous surjection $f : X \to Y$ is said to be *d-open* if the image $f[U]$ is dense in a non-empty open subset $V \subseteq Y$, whenever $U \subseteq X$ is open and non-empty, i.e. $f[U] \subseteq V$ and $\mathrm{cl}_Y f[U] = \mathrm{cl}_Y V$. The notion of d-open maps was introduced by M. G. Tkachenko 1981.

A function

$$f : \mathbb{R} \times \{0\} \cup \mathbb{Q} \times \{1\} \to \mathbb{R},$$

defined by the formula $f(x, i) = x$ for any $x \in \mathbb{R}, i \in \{0, 1\}$ is an example of d-open but not open map.

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Main Theorem

We say that a space X is *an almost limit* of the inverse system $S = \{X_\sigma, \pi^\sigma_\varrho, \Sigma\}$, if X can be embedded in $\varprojlim S$ such that $\pi_\sigma(X) = X_\sigma$ for each $\sigma \in \Sigma$. We denote this by $X = \mathrm{a} - \varprojlim S$, and it implies that X is a dense subset of $\varprojlim S$.

Theorem

If X is pseudocompact countable κ-metrizable space, then

$$X = a - \varprojlim \{X_\sigma, \pi^\sigma_\varrho, \Sigma\},$$

where $\{X_\sigma, \pi^\sigma_\varrho, \Sigma\}$ is a σ-complete inverse system, all spaces X_σ are compact and metrizable, and all bonding maps π^σ_ϱ are open.
Moreover the space $Y = \varprojlim \{X_\sigma, \pi^\sigma_\varrho, \Sigma\}$ is Čech-Stone compactification of X.

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Corollary

If X is pseudocompact conutable κ-metrizable space, then X satisfies c.c.c.

Example

ω_1 *with the order topology is pseudocompact and not countable κ-metrizable.*

If ω_1 will be countable κ-metrizable then by Corollary it satisfies c.c.c., a contradiction.

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A. Chigogidze was announced (1982) without proof that Čech-Stone compactification of pseudocompact κ-metrizable space is κ-metrizable.

Next G. Dimov gave (1983) sufficient and necessary conditions that compact Hausdorff extension of κ-metrizable space is κ-metrizable.

We give a simple proof that Čech-Stone compactification of pseudocompact countable κ-metrizable space is κ-metrizable.

Assume that X is pseudocompact countable κ-metrizable space. We can extend each function $\rho(\cdot, F \cap X)$ to continuous function $\bar{\rho}(\cdot, F \cap X) : \beta X \to [0, \infty)$, where $F \in \mathrm{RC}(\beta X)$.

Next we can prove that $\bar{\rho}$ satisfies condition $(K1) - (K3)$ and $(K4_\omega)$. Since pseudocompact countable κ-metrizable space satisfies c.c.c. $\bar{\rho}$ is κ-metric.

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