Generic objects in topology

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The result is an inverse sequence

$$\vec{k} = \langle K_i, k_i^j, \omega \rangle.$$

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The game above will be called the Banach-Mazur game with parameters $\mathfrak{K}$ and U.

Uniqueness

Proposition

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2. Assume Odd uses Σ_0.
3. Assume Eve uses Σ_1.
4. They both win!
5. Thus $U_0 \approx U_1$.

□

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Let U be a $\mathfrak{K}$-generic space. Then for every $K \in \mathfrak{K}$ there exists a continuous surjection $q \colon U \to K$.
If all $\mathfrak{K}$-arrows are retractions, then q is a retraction.

Basic examples

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Example

Let Fin^+ be the category of nonempty finite sets with surjections. Then 2^ω is Fin^+-generic.

Continua

Proposition

There is no generic continuum.
More precisely, there is no $\mathfrak{C}$-generic space, where $\mathfrak{C}$ is the category of continua with continuous surjections.

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Proof.

Use Waraszkiewicz spirals. □

The pseudo-arc

Notation:

Denote by $\mathfrak{I}$ the category whose unique object is the unit interval $\mathbb{I} = [0, 1]$ and arrows are continuous surjections.

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Theorem

There exists an $\mathfrak{I}$-generic continuum, namely, the pseudo-arc $\mathbb{P}$.

Domination

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2. For every $\varepsilon > 0$, for every $\mathfrak{K}$-arrow $p\colon X \to X_0$ with $X_0 \in \text{Obj}(\mathfrak{K}_0)$ there exists a $\mathfrak{K}$-arrow $q\colon Y_0 \to X$ with $Y_0 \in \text{Obj}(\mathfrak{K}_0)$ such that $p \circ q$ is ε-close to some $\mathfrak{K}_0$-arrow $g\colon Y_0 \to X_0$.

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$$(\forall\, y \in Y_0) \quad \varrho(p(q(y)), g(y)) \leqslant \varepsilon,$$

where ϱ is a fixed metric on X_0.

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Use Mioduszewski's result (1963) on homeomorphisms of inverse limits. □

Corollary

The pseudo-arc $\mathbb{P}$ is generic in the class of all Peano continua.

Fix a category $\mathfrak{C} \subseteq \mathfrak{CM}^+$ whose arrows are surjections.

Definition

We say that $\mathfrak{C}$ is directed if for every $X, Y \in \mathrm{Obj}(\mathfrak{C})$ there exist $W \in \mathrm{Obj}(\mathfrak{C})$ and $\mathfrak{C}$-arrows $f\colon W \to X$, $g\colon W \to Y$.

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Definition

We say that $\mathfrak{C}$ has the almost amalgamation property if for every $\varepsilon > 0$, for every $\mathfrak{C}$-arrows $f\colon X \to Z$, $g\colon Y \to Z$, there exist $\mathfrak{C}$-arrows $f'\colon W \to X$ and $g'\colon W \to Y$ such that the diagram

$$\begin{array}{ccc} Y & \xleftarrow{\;g'\;} & W \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle f'} \\ Z & \xleftarrow[\;f\;]{} & X \end{array}$$

is ε-commutative.

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Theorem

Assume $\mathfrak{K} \subseteq \mathfrak{CM}^+$ and $\mathfrak{K}$-arrows are surjections.
Suppose $\mathfrak{K}$ contains a dominating directed subcategory with the almost amalgamation property and with countably many objects.
Then there exists a $\mathfrak{K}$-generic object.

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Definition

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1. For every $X \in \mathrm{Obj}(\mathfrak{K})$ there are $n \in \omega$ and a $\mathfrak{K}$-arrow $f \colon U_n \to X$.
2. For every $\varepsilon > 0$, for every $n \in \omega$, for every $\mathfrak{K}$-arrow $f \colon Y \to U_n$, there exist $m > n$ and a $\mathfrak{K}$-arrow $g \colon U_m \to Y$ that is ε-close to the bonding arrow $u_n^m \colon U_m \to U_n$.

Main results

Theorem

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Let $\vec{u}$ be a Fraïssé sequence in $\mathfrak{K}$, $U = \varinjlim \vec{u}$. Then U is $\mathfrak{K}$-generic.

Back to the pseudo-arc

Lemma

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Use the Mountain Climbing Theorem. □

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Theorem

Let $\vec{u}$ be a Fraïssé sequence in $\mathfrak{I}$. Then $\varprojlim \vec{u}$ is the pseudo-arc.

Corollary (Irwin & Solecki 2006)

Let P be a chainable continuum. Then P is homeomorphic to the pseudo-arc if and only if for every $\varepsilon > 0$ for every continuous surjections $f\colon P \to \mathbb{I}$, $g\colon \mathbb{I} \to \mathbb{I}$ there exists a continuous surjection $h\colon P \to \mathbb{I}$ such that $g \circ h$ is ε-close to f.

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Proof.

Use Cook's continuum. For details, see

A. Całka, *Skracanie produktów topologicznych*, MSc, Uniwersytet Warszawski, 2008.

□

Example

Fix $n \in \omega \cup \{\infty\}$. Let $\mathfrak{D}_n$ ($\mathfrak{D}^c_n$) be the category whose objects are nonempty (connected) polyhedra of dimension $\leqslant n$, and arrows are continuous retractions.

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Corollary

There exist a $\mathfrak{D}_n$-generic and a $\mathfrak{D}_n^c$-generic space.

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Let $\mathfrak{S}$ be the category whose objects are finite-dimensional simplices and arrows are affine surjections.

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$\mathfrak{S}$ is a compact Fraïssé category and its Fraïssé limit is the Poulsen simplex, the unique metrizable simplex whose set of extreme points is everywhere dense.

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Parallel results concerning the Lelek fan:

- A. Kwiatkowska, W. Kubiś, *The Lelek fan and the Poulsen simplex as Fraïssé limits*, preprint, 2015.

Embeddings as Fraïssé limits

Example (W. Bielas, M. Walczyńska, W.K.)

Fix a compact 0-dimensional metric space $A \neq \emptyset$. Consider the following category $\mathfrak{F}_A$.

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Fix a compact 0-dimensional metric space $A \neq \emptyset$. Consider the following category $\mathfrak{F}_A$.
The objects are continuous mappings $f\colon A \to s$, where s is finite. An arrow from $f\colon A \to s$ to $g\colon A \to t$ is a continuous surjection $p\colon t \to s$ such that $p \circ g = f$.

$$\begin{array}{ccc} t & \xleftarrow{g} & A \\ & \overset{p}{\searrow\!\!\searrow} & \downarrow f \\ & & s \end{array}$$

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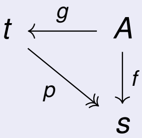

Lemma

$\mathfrak{F}_A$ is a Fraïssé category.

Theorem (W. Bielas, M. Walczyńska, W.K.)

Let $e\colon A \to 2^\omega$ be a topological embedding such that $e[A]$ is nowhere dense in 2^ω. Then e is the Fraïssé limit of $\mathfrak{F}_A$.

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Corollary (Knaster & Reichbach)

Every homeomorphism between two closed nowhere dense subsets of 2^ω extends to an auto-homeomorphism of 2^ω.

References

W. Kubiś, *Metric-enriched categories and approximate Fraïssé limits*, preprint, http://arxiv.org/abs/1210.6506