Near-rings of Continuous Functions and Primeness

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1. Preliminaries

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Definition 1.1

A (right) near-ring is a triple $(N, +, \cdot)$ where

1. $(N, +)$ is a (not necessarily Abelian) group;
2. $(N, \cdot)$ is a semigroup;
3. $(x + y)z = xz + yz$ for all $x, y, z \in N$.
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A normal subgroup I of $(N, +)$ is called a left ideal of N if
$r(x + s) - rs \in I$ for all $r, s \in N$ and $x \in I$.
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As we shall see, the instances where $N_0(G)$ is simple seem to be the exception rather than the rule.

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Definition 2.2

Let $(G, +)$ be a topological group. Suppose that for every proper closed subset F of G, $x \in G \backslash F$ and $0 \neq y \in G$, there exists a continuous function $f : G \to G$ such that $f(F) = 0$ and $f(x) = y$ Then G is called an S^*-group.

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Let $(G, +)$ be an S^*-group or be disconnected. Then $N_0(G)$ is simple if and only if the topology on G is discrete (Magill, 1967).

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It seems easier to find cases where $N(G)$ is simple, as the next result shows.

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Let $G := \mathbb{R} \times \mathbb{Z}_2$, where G has the product topology with respect to the usual and discrete topologies on $\mathbb{R}$ and $\mathbb{Z}_2$, respectively. Let $I := \{a \in N_0(G) : a(\mathbb{R} \times 0) = 0\}$ and $J := \{a \in N_0(G) : a(G) \subseteq \mathbb{R} \times 0\}$. Then I and J are ideals of $N_0(G)$ and $I \cap J \neq 0$. However, $(I \cap J)^2 = 0$, so $N_0(G)$ is not 0-prime and hence not equiprime.

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The different notions of primeness discussed in this talk give rise to different prime radicals. If N is a near-ring and $\nu \in \{0, 3.e\}$, let

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Let G be a disconnected topological group, with open components which each contain more than one element. Let H be the component of G which contains 0. Let $I := \{a \in N_0(G) : a(H) = 0\}$ and $J := \{a \in N_0(G) : a(G) \subseteq H\}$. Then $\mathcal{P}_0(N_0(G)) = \mathcal{P}_3(N_0(G)) = \mathcal{P}_e(N_0(G)) = I \cap J$.

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Let G be an arcwise connected topological group with more than one element. Then $N_0(G)$ is not strongly prime (Booth and Hall, 2004).

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An ideal I of a near-ring N is strongly prime (resp. uniformly strongly prime) if the factor near-ring N/I is strongly prime, (resp. uniformly strongly prime).

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An ideal I of a near-ring N is strongly prime (resp. uniformly strongly prime) if the factor near-ring N/I is strongly prime, (resp. uniformly strongly prime). The strongly prime radical $\mathcal{P}_s(N)$ (resp. uniformly strongly prime radical $\mathcal{P}_u(N)$) is the intersection of the strongly prime (resp. uniformly strongly prime) ideals of N.

4. Strongly Prime Near-rings

Theorem 4.1

Let G be a 0-dimensional topological group. Then

1. *$N_0(G)$ is strongly prime if and only if the topology on G is discrete;*
2. *$N_0(G)$ is strongly equiprime if and only if G is finite.*
   (Booth and Hall, 2004)

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$P_{\mathbb{R}^n} = \{a \in N_0(\mathbb{R}^n)$: *there exists a neighbourhood* U *of* 0 *such that* $a(U) = 0\}$ *is a uniformly strongly prime ideal of* $N_0(\mathbb{R}^n)$ *which contains every strongly prime ideal (Booth, 2010).*

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Corollary 4.3

$\mathcal{P}_s(N_0(\mathbb{R}^n)) = \mathcal{P}_u(N_0(\mathbb{R}^n)) = P_{\mathbb{R}^n}.$

4. Strongly Prime Near-rings

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Now we investigate strongly prime ideals in $N_0(\mathbb{R}^\omega)$, where ω is the first transfinite cardinal, and $\mathbb{R}^\omega$ has the usual (Tychonoff) product topology.

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$d(x, y) := \sum_{i=1}^{\infty} \frac{|x_i - y_i|}{2^i(1+|x_i-y_i|)}$, where $x := (x_i)_{i\in\mathbb{N}}$ and $y := (y_i)_{i\in\mathbb{N}}$.

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At this stage, we have not been able to characterise $\mathcal{P}_u(N_0(\mathbb{R}^\omega))$.

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Thank you!

Děkuji!