Chaos in hyperspaces of nonautonomous discrete systems

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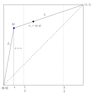
Twelfth Symposium on General Topology and its Relations to Modern Analysis and Algebra
2016



Hyperspaces of continua

Definition

Given a topological space X, let $f_n : X \to X$ be a continuous function for each positive integer n. Denote by f_∞ the sequence $(f_1, f_2, \ldots)$. We say that the pair (X, f_∞) is the *nonautonomous discrete dynamical system* (NDS, for short) in which the *orbit of a point* $x \in X$ under f_∞ is defined as the set

$$\operatorname{orb}(x, f_\infty) = \{x, f_1(x), f_1^2(x), \ldots, f_1^n(x), \ldots\},$$

where

$$f_1^n := f_n \circ f_{n-1} \circ \cdots \circ f_2 \circ f_1,$$

for each positive integer n.

In particular, when f_∞ is the constant sequence $(f, f, \ldots, f, \ldots)$, the pair (X, f_∞) is the usual (autonomous) discrete dynamical system given by the continuous function f on X and it will denoted by (X, f).

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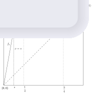
Definitions

Definition

A NDS (X, f_∞) is

- *topologically transitive* if for any two non-empty open sets U and V in X, there exists a positive integer k such that $f_1^k(U) \cap V \neq \emptyset$;
- said to satisfy *Banks' condition* if for any three non-empty open sets U, V, W in X, there exists a positive integer k such that $f_1^k(U) \cap V \neq \emptyset$ and $f_1^k(U) \cap W \neq \emptyset$;
- *weakly mixing* if for any four non-empty open sets U_1, U_2, V_1, V_2 in X, there exists a positive integer k such that $f_1^k(U_i) \cap V_i \neq \emptyset$, for each $i \in \{1, 2\}$.

If (X, f_∞) is weakly mixing, then it has Banks' condition and, this implies that it is transitive.



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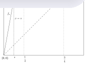

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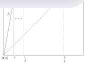

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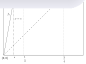

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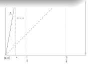
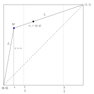


Definitions

Let X be a topological space. The symbol $\mathcal{K}(X)$ will denote the hyperspace of all non-empty compact subsets of X endowed with the Vietoris topology.

Induced NDS

Given a continuous function $f : X \to X$, it induces a continuous function on $\mathcal{K}(X)$, $\overline{f} : \mathcal{K}(X) \to \mathcal{K}(X)$ defined by $\overline{f}(K) = f(K)$ for every $K \in \mathcal{K}(X)$.
Let (X, f_∞) be a NDS and $\overline{f_n}$ the induced continuous function of f_n on $\mathcal{K}(X)$, for each positive integer n. Then, the sequence $\overline{f_\infty} = (\overline{f_1}, \overline{f_2}, \ldots, \overline{f_n}, \ldots)$ induces a nonautonomous discrete dynamical system $(\mathcal{K}(X), \overline{f_\infty})$. In this case, $\overline{f}_1^n = \overline{f_n} \circ \cdots \circ \overline{f_2} \circ \overline{f_1}$.



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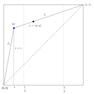

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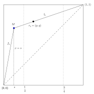


Theorem. (Peris, 2004)

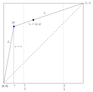
Let $f\colon X \to X$ be a continuous function on a topological space X. Then the following conditions are equivalent:

(1) (X, f) is weakly mixing.

(2) $(\mathcal{K}(X), \overline{f})$ is weakly mixing.

(3) $(\mathcal{K}(X), \overline{f})$ is transitive.

(2) implies (3) even in nonautonomous discrete dynamical systems.



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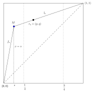

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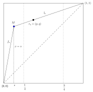

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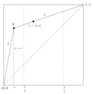

Example

There is a NDS $(\mathbb{I}, f_\infty)$ which is weakly mixing, but $(\mathcal{K}(\mathbb{I}), \overline{f_\infty})$ is not transitive.

Let
$F = \{(a,b,c,d) \in \mathbb{Q}^4 : a,b,c,d \in (0,1), a < b, a \neq c, b \neq d, c \neq d\}$.
Clearly, F is countable. We will assign a homeomorphism $f : \mathbb{I} \to \mathbb{I}$ to every element $(a,b,c,d) \in F$ as follows:
Case 1. If $c < d$, then f is the function whose graphic is determined by the segments $[(0,0),(a,c)]$, $[(a,c),(b,d)]$ and $[(b,d),(1,1)]$.
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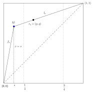

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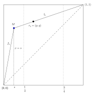

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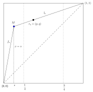

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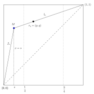

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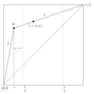

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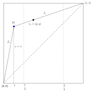

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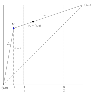

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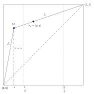

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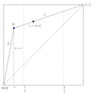

Results

Banks proved that in autonomous discrete dynamical systems the property of being weakly mixing is equivalent to satisfy Banks' condition.

Theorem

If $(\mathcal{K}(X), \overline{f_\infty})$ is transitive, then (X, f_∞) satisfies Banks' condition.

Proposition

If $(\mathcal{K}(X), \overline{f_\infty})$ is transitive, then so is (X, f_∞).

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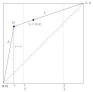

Results

Banks proved that in autonomous discrete dynamical systems the property of being weakly mixing is equivalent to satisfy Banks' condition.

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If $(\mathcal{K}(X), \overline{f_\infty})$ is transitive, then (X, f_∞) satisfies Banks' condition.

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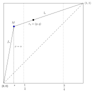

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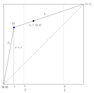

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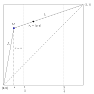

Results

Definition

We say that (X, f_∞) is *weakly mixing of order m* $(m \geq 2)$ if for any non-empty open sets $U_1, U_2, \ldots, U_m, V_1, V_2, \ldots, V_m$ in X, there exists a positive integer k such that $f_1^k(U_i) \cap V_i \neq \emptyset$ for each $i \in \{1, 2, \ldots, m\}$.

Theorem

Suppose that $(\mathcal{K}(X), \overline{f_\infty})$ is weakly mixing of order m. Then so is (X, f_∞).

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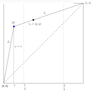
$(\mathbb{I}, f_\infty)$ is weakly mixing of order 3 if and only if $(\mathcal{K}(\mathbb{I}), \overline{f_\infty})$ is weakly mixing of order 3.



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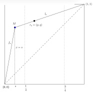

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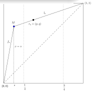

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Given a NDS (X, f_∞), a point $x \in X$ is *periodic* if $f_1^n(x) = x$ for some positive integer n. Let us denote by $Per(f_\infty)$ the *set of periodic points* of f_∞.

Definition

Let (X, d) be a metric space. We say that (X, f_∞) has sensitive dependence on initial conditions if there exists $\delta > 0$ such that for every point x and every open neighborhood U of x, there exist $y \in U$ and $n \in \mathbb{N}$ such that $d(f_1^n(x), f_1^n(y)) \geq \delta$.

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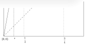
Given a metric space X we say that the NDS (X, f_∞) is Devaney chaotic if it is transitive, sensitive and has dense set of periodic points.



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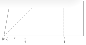

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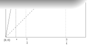


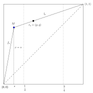
Results

Theorem

Let (X, d) be a compact metric space. If $(\mathcal{K}(X), \overline{f_\infty})$ has sensitive dependence on initial conditions, then (X, f_∞) does.

Example

There is a NDS $(\mathbb{I}, f_\infty)$ which is transitive and has dense set of periodic points, but it does not have sensitive dependence on initial conditions.



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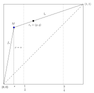

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A NDS (X, f_∞) is said to be *point transitive* if there exists $x \in X$ with dense orbit in X.

Proposition

If $(\mathcal{K}(X), \overline{f_\infty})$ is point transitive, then so is (X, f_∞).

It is known that point transitivity is equivalent to transitivity for autonomous discrete dynamical systems on complete separable metric spaces without isolated points.

Proposition

Suppose that X is a second-countable space with the Baire property. If (X, f_∞) is transitive, then it is point transitive.

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There is a NDS $(\mathbb{I}, g_\infty)$ which is point transitive but it is not transitive.

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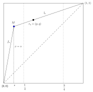
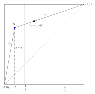
M. Vellekoop and R. Berglund showed (1994) that for autonomous discrete dynamical systems on the unit interval $\mathbb{I}$ to be Devaney chaotic is equivalent to be transitive

Example

There is a transitive NDS $(\mathbb{I}, g_\infty)$ with sensitive dependence on initial conditions such that the set of periodic points is not dense in $\mathbb{I}$.





Thank you!