# EXAMPLES OF ABSORBERS IN CONTINUUM THEORY

Paweł Krupski and Alicja Samulewicz

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 $2^X = \{A \subset X : A \text{ is a closed subset of } X\}$ 

 $C(X) = \{A \subset X : A \text{ is a continuum}\}.$ 

 $\mathcal{H}$  is a hyperspace of X if  $\mathcal{H} \subseteq 2^X$ .

Hyperspaces of a continuum (X, d) are equipped with the Hausdorff metric  $d_H$ .

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## THEOREM (CURTIS, SCHORI)

If X is a Peano continuum then its hyperspace  $2^X$  is homeomorphic to the Hilbert cube  $I^{\omega}$ . If X is a Peano continuum without free arcs then the hyperspace C(X) is homeomorphic to  $I^{\omega}$  as well. *X* is a *space without free arcs* provided that all arcs in a space *X* have empty interiors.

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- $A \cap f[X] = \emptyset$
- $d_{sup}(f, id_X) < \varepsilon$ .

A countable union of Z-sets in X is called a  $\sigma$ Z-set in X.

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Let  ${\mathcal M}$  be a Borel or a projective class.

A subset *D* of a Hilbert cube *X* is an  $\mathcal{M}$ -**absorber** in *X* provided that

•  $D \in \mathcal{M};$ 

D is contained in a  $\sigma Z$ -set in X;

D is strongly M-universal, i.e., for each subset M ∈ M of I<sup>ω</sup> and for each compact set K ⊂ I<sup>ω</sup>, any embedding f : I<sup>ω</sup> → X such that f(K) is a Z-set in X can be approximated arbitrarily closely (in the "sup" metric d) by an embedding g : I<sup>ω</sup> → X such that g(I<sup>ω</sup>) is a Z-set in X, g|K = f|K and g<sup>-1</sup>(D) \ K = M \ K.

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# TOPOLOGICAL UNIQUENESS

## THEOREM

If  $A \subset X$  and  $B \subset Y$  are  $\mathcal{M}$ -absorbers in Hilbert cubes X and Y, respectively, then there exists a homeomorphism  $h : X \to Y$  with h[A] = B.

#### THEOREM

For any Borel (except for  $G_{\delta}$ ) or projective class  $\mathcal{M}$  exists a set  $M \subset I^{\omega}$  that is an  $\mathcal{M}$ -absorber in  $I^{\omega}$ . Moreover, there exists an incomplete linear subspace of  $I_2$  homeomorphic to the  $\mathcal{M}$ -absorber M.

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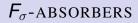
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# **PROPERTIES OF ABSORBERS**

# COROLLARY

All absorbers in the Hilbert cube are

- homogeneous
- arcwise connected
- not complete metrizable
- not locally compact.



# A standard $F_{\sigma}$ -absorber in the Hilbert cubes $I^{\omega}$ is its pseudoboundary

$$B(I^{\omega}) = \{(x_i) \in I^{\omega} : \exists i \ x_i \in \{0, 1\} \}.$$

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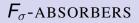
- the family of compacta with nonempty interiors in  $2^X$ , where X is a Peano continuum (Curtis, Michael, 1987)
- Oim<sub>≥n</sub> in the Hilbert cube  $2^{l^{\omega}}$ , n ≥ 1 (Dijkstra, van Mill, Mogilski, 1992)
- Oim<sub>≥n</sub> in the Hilbert cube 2<sup>X</sup>, where X is a Peano continuum each of whose open non-empty subset has dimension ≥ n, n ≥ 1 (Cauty, 1999)
- the family D(I<sup>n</sup>) of all decomposable subcontinua of I<sup>n</sup>, n ≥ 3, in the Hilbert cube C(I<sup>n</sup>) (A.S., 2008)
- the family of compacta that block all subcontinua of a Peano continuum which is not separated by any finite set (Illanes, P. Krupski, 2011)

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# A closed subset C is a *separator* in a space X if $X \setminus C$ is disconnected.

S(X) – the family of all closed separators of X.

### Theorem

# Let X be a Peano continuum such that

- each open non-empty subset of X contains a copy of (0,1)<sup>n</sup>, 3 ≤ n < ∞, as an open subset,</li>
- no subset of dimension  $\leq$  1 separates *X*.

Then the families S(X) and  $S(X) \cap C(X)$  are  $F_{\sigma}$ -absorbers in  $2^X$  and C(X), respectively.

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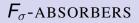
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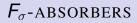
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## COROLLARY

If a continuum X is an n-manifold (with or without boundary),  $3 \le n < \infty$ , then S(X) and  $S(X) \cap C(X)$  are  $F_{\sigma}$ -absorbers in  $2^X$ and C(X), respectively. In particular,  $S(X) \cong S(X) \cap C(X) \cong B(I^{\omega})$ .

# $D_2(F_{\sigma})$ -Absorbers

The Borel class  $D_2(F_{\sigma})$  consists of all sets that are differences of two  $F_{\sigma}$ -sets.

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A standard  $D_2(F_{\sigma})$ -absorber in  $I^{\omega} \times I^{\omega}$  is  $B(I^{\omega}) \times (0,1)^{\omega}$ .

# $\mathcal{N}(X)$ – the family of all nowhere dense closed subsets of XS(X) – the family of all closed separators of X

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Assume X is a Peano continuum such that

each open non-empty subset of X contains a copy of (0,1)<sup>n</sup>, 3 ≤ n < ∞, as an open subset,</li>

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Then  $S(X) \cap \mathcal{N}(X)$  is a  $D_2(F_{\sigma})$ -absorber in  $2^X$  and  $S(X) \cap \mathcal{N}(X) \cap C(X)$  is a  $D_2(F_{\sigma})$ -absorber in C(X).

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# $S(X)_{n-1}$ – the family of all (n-1)-dimensional closed separators of X

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If a continuum X is an n-manifold (with or without boundary),  $3 \le n < \infty$ , then

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In particular,  $S(X)_{n-1} \cong S(X)_{n-1} \cap C(X) \cong B(I^{\omega}) \times (0,1)^{\omega}$ .

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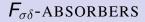
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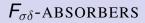
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Standard  $F_{\sigma\delta}$ -absorbers:

• 
$$(B(I^{\omega}))^{\omega}$$
 in  $(I^{\omega})^{\omega}$ ,

• 
$$\widehat{c}_0 = \{(x_i) \in I^\omega : \lim_i x_i = 0\}$$
 in  $I^\omega$ .

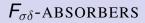


Examples of  $F_{\sigma\delta}$ -absorbers:

- the family of all infinite-dimensional compact subsets of  $I^{\omega}$  (Dijkstra, van Mill, Mogilski, 1992)
- *LC*(*I<sup>n</sup>*) − the family of all locally connected subcontinua of *I<sup>n</sup>*, *n* ≥ 3 (Gladdines, van Mill, 1993)

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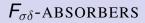


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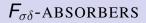


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*X* is *aposyndetic*  $\Leftrightarrow$  each  $x \in X$  has an arbitrarily small neighbourhood *U* such that  $X \setminus U$  has finitely many components.

Col(X) – the family of colocally connected continua in X Apo(X) – the family of aposyndetic continua in X

#### Theorem

If  $n \ge 3$  then Apo( $I^n$ ) and Col( $I^n$ ) are  $F_{\sigma\delta}$ -absorbers in C( $I^n$ ).

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*X* is a *Kelley continuum*  $\Leftrightarrow$  for each  $x \in X$ , each sequence  $x_n \to x$  and each  $Z \in C(X)$  with  $x \in Z$ , there are  $Z_n \in C(X)$  such that  $x_n \in Z_n$  and  $d_H(Z_n, Z) \to 0$ .

 $\mathcal{K}(X)$  – the family of Kelley continua in X

Theorem

If  $n \geq 3$  then  $\mathcal{K}(I^n) \cap \mathcal{D}_2(I^n)$  and  $\mathcal{K}(I^n) \cap \text{Decomp}(I^n)$  are  $F_{\sigma\delta}$ -absorbers in  $\mathcal{C}(I^n)$ .

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## THEOREM (CAUTY, 1991)

The Hurewicz set  $\mathcal{H} = \{A \in 2^{I} : |A| \le \omega\}$  is a coanalytic absorber in  $2^{I}$ .

### Theorem (Cauty, 1991)

The space of all differentiable functions  $f : I \to \mathbb{R}$  is homeomorphic to the Hurewicz set  $\mathcal{H}$ .

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*Y* is *strongly countable dimensional* if *Y* is a countable union of its compact, finite-dimensional subspaces.

*X* is *strongly infinite-dimensional* if there exists a sequence  $(A_n, B_n)_n$  of closed disjoint subsets of *X* such that for each sequence  $(C_n)_n$  of closed separators of *X* between  $A_n$  and  $B_n$  we have  $\bigcap_n C_n \neq \emptyset$ .

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# $SCD_n(X)$ – the family of all strongly countable-dimensional compacta of dimension $\ge n$ in X.

 $W_n(X)$  – the family of all weakly infinite-dimensional compacta of dimension  $\ge n$  in X.

### Theorem

Let X be a locally connected continuum such that each non-empty open subset of X contains a copy of the Hilbert cube.

- $SCD_n(X)$  and  $W_n(X)$  are coanalytic absorbers in  $2^X$  for  $n \ge 1$ .
- SCD<sub>n</sub>(X) ∩ C(X) and W<sub>n</sub>(X) ∩ C(X) are coanalytic absorbers in C(X) for n ≥ 2.

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# *X* is *hereditarily locally connected* = each subcontinuum of *X* is locally connected.

 $\mathcal{HLC}(X)$  – the family of all hereditarily locally connected subcontinua of X

X is Suslinian = each collection of pairwise disjoint nondegenerate subcontinua of X is countable.

Susl(X) – the family of all Suslinian subcontinua of X

### Theorem (Darji, Marcone, 2004)

If  $n \ge 2$  then the families  $\mathcal{HLC}(I^n)$  and  $SusI(I^n)$  are coanalytic complete.

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#### THEOREM

# Thank you!

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