EXAMPLES OF ABSORBERS IN CONTINUUM THEORY

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PEANO CONTINUA

Continuum – compact connected metric space.

A continuum is *locally connected* if its every point has arbitrarily small connected open neighbourhoods.

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HYPERSPACES

$2^X = \{A \subset X : A \text{ is a closed subset of } X\}$

$C(X) = \{A \subset X : A \text{ is a continuum}\}$.

$\mathcal{H}$ is a *hyperspace* of X if $\mathcal{H} \subseteq 2^X$.

Hyperspaces of a continuum (X, d) are equipped with the Hausdorff metric d_H.

X is a *space without free arcs* provided that all arcs in a space X have empty interiors.

THEOREM (CURTIS, SCHORI)

If X is a Peano continuum then its hyperspace 2^X is homeomorphic to the Hilbert cube I^ω.
If X is a Peano continuum without free arcs then the hyperspace $C(X)$ is homeomorphic to I^ω as well.

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σZ-SETS

A closed subset A of a Hilbert cube X is a *Z-set* in X if for every $\varepsilon > 0$ exists a continuous mapping $f : X \to X$ such that

- $A \cap f[X] = \emptyset$
- $d_{sup}(f, id_X) < \varepsilon$.

A countable union of Z-sets in X is called a σZ*-set* in X.

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Absorbers in Hilbert cubes

Let $\mathcal{M}$ be a Borel or a projective class.

A subset D of a Hilbert cube X is an $\mathcal{M}$-**absorber** in X provided that

1. $D \in \mathcal{M}$;
2. D is contained in a σZ-set in X;
3. D is strongly $\mathcal{M}$-universal, i.e., for each subset $M \in \mathcal{M}$ of I^ω and for each compact set $K \subset I^\omega$, any embedding $f : I^\omega \to X$ such that $f(K)$ is a Z-set in X can be approximated arbitrarily closely (in the “sup" metric $\widetilde{d}$) by an embedding $g : I^\omega \to X$ such that $g(I^\omega)$ is a Z-set in X, $g|K = f|K$ and $g^{-1}(D) \setminus K = M \setminus K$.

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Topological uniqueness

Theorem

If $A \subset X$ and $B \subset Y$ are $\mathcal{M}$-absorbers in Hilbert cubes X and Y, respectively, then there exists a homeomorphism $h : X \to Y$ with $h[A] = B$.

Theorem

For any Borel (except for G_δ) or projective class $\mathcal{M}$ exists a set $M \subset I^\omega$ that is an $\mathcal{M}$-absorber in I^ω.
Moreover, there exists an incomplete linear subspace of l_2 homeomorphic to the $\mathcal{M}$-absorber M.

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Properties of absorbers

Corollary

All absorbers in the Hilbert cube are

- *homogeneous*
- *arcwise connected*
- *not complete metrizable*
- *not locally compact.*

F_σ-ABSORBERS

A standard F_σ-absorber in the Hilbert cubes I^ω is its pseudoboundary

$$B(I^\omega) = \{(x_i) \in I^\omega : \exists i \quad x_i \in \{0, 1\}\}.$$

F_σ-ABSORBERS

Examples of F_σ-absorbers in Hilbert cubes:

1. the family of compacta with nonempty interiors in 2^X, where X is a Peano continuum (Curtis, Michael, 1987)
2. $Dim_{\geq n}$ in the Hilbert cube 2^{I^ω}, $n \geq 1$ (Dijkstra, van Mill, Mogilski, 1992)
3. $Dim_{\geq n}$ in the Hilbert cube 2^X, where X is a Peano continuum each of whose open non-empty subset has dimension $\geq n$, $n \geq 1$ (Cauty, 1999)
4. the family $\mathcal{D}(I^n)$ of all decomposable subcontinua of I^n, $n \geq 3$, in the Hilbert cube $C(I^n)$ (A.S., 2008)
5. the family of compacta that block all subcontinua of a Peano continuum which is not separated by any finite set (Illanes, P. Krupski, 2011)

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F_σ-ABSORBERS

A closed subset C is a *separator* in a space X if $X \setminus C$ is disconnected.

$S(X)$ – the family of all closed separators of X.

THEOREM

Let X be a Peano continuum such that

- *each open non-empty subset of X contains a copy of $(0,1)^n$, $3 \le n < \infty$, as an open subset,*
- *no subset of dimension ≤ 1 separates X.*

Then the families $\mathcal{S}(X)$ and $\mathcal{S}(X) \cap C(X)$ are F_σ-absorbers in 2^X and $C(X)$, respectively.

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F_σ-ABSORBERS

COROLLARY

If a continuum X is an n-manifold (with or without boundary), $3 \leq n < \infty$, then $\mathcal{S}(X)$ and $\mathcal{S}(X) \cap C(X)$ are F_σ-absorbers in 2^X and $C(X)$, respectively.
In particular, $\mathcal{S}(X) \cong \mathcal{S}(X) \cap C(X) \cong B(I^\omega)$.

$D_2(F_\sigma)$-ABSORBERS

The Borel class $D_2(F_\sigma)$ consists of all sets that are differences of two F_σ-sets.

A standard $D_2(F_\sigma)$-absorber in $I^\omega \times I^\omega$ is $B(I^\omega) \times (0,1)^\omega$.

$D_2(F_\sigma)$-ABSORBERS

$\mathcal{N}(X)$ – the family of all nowhere dense closed subsets of X

$S(X)$ – the family of all closed separators of X

Theorem

Assume X is a Peano continuum such that

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Then $\mathcal{S}(X) \cap \mathcal{N}(X)$ is a $D_2(F_\sigma)$-absorber in 2^X and $\mathcal{S}(X) \cap \mathcal{N}(X) \cap C(X)$ is a $D_2(F_\sigma)$-absorber in $C(X)$.

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$\mathcal{S}(X)_{n-1}$ – the family of all $(n-1)$-dimensional closed separators of X

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In particular, $\mathcal{S}(X)_{n-1} \cong \mathcal{S}(X)_{n-1} \cap C(X) \cong B(I^\omega) \times (0,1)^\omega$.

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Standard $F_{\sigma\delta}$-absorbers:

- $(B(I^\omega))^\omega$ in $(I^\omega)^\omega$,
- $\widehat{c_0} = \{(x_i) \in I^\omega : \lim_i x_i = 0\}$ in I^ω.

Examples of $F_{\sigma\delta}$-absorbers:

1. the family of all infinite-dimensional compact subsets of I^ω (Dijkstra, van Mill, Mogilski, 1992)
2. $\mathcal{LC}(I^n)$ – the family of all locally connected subcontinua of I^n, $n \geq 3$ (Gladdines, van Mill, 1993)
3. the family of all continua being absolute retracts in I^2 (Cauty, Dobrowolski, Gladdines, van Mill, 1995).

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$F_{\sigma\delta}$-ABSORBERS

X is *colocally connected* $\Leftrightarrow$ each $x \in X$ has an arbitrarily small neighbourhood U such that $X \setminus U$ is connected.

X is *aposyndetic* $\Leftrightarrow$ each $x \in X$ has an arbitrarily small neighbourhood U such that $X \setminus U$ has finitely many components.

$Col(X)$ – the family of colocally connected continua in X
$Apo(X)$ – the family of aposyndetic continua in X

THEOREM

If $n \geq 3$ then $Apo(I^n)$ and $Col(I^n)$ are $F_{\sigma\delta}$-absorbers in $C(I^n)$.

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X is a *Kelley continuum* $\Leftrightarrow$ for each $x \in X$, each sequence $x_n \to x$ and each $Z \in C(X)$ with $x \in Z$, there are $Z_n \in C(X)$ such that $x_n \in Z_n$ and $d_H(Z_n, Z) \to 0$.

$\mathcal{K}(X)$ – the family of Kelley continua in X

THEOREM

If $n \geq 3$ then $\mathcal{K}(I^n) \cap \mathcal{D}_2(I^n)$ and $\mathcal{K}(I^n) \cap Decomp(I^n)$ are $F_{\sigma\delta}$-absorbers in $C(I^n)$.

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Π^1_1-absorbers (coanalytic absorbers)

Theorem (Cauty, 1991)

The Hurewicz set $\mathcal{H} = \{A \in 2^I : |A| \leq \omega\}$ *is a coanalytic absorber in* 2^I.

Theorem (Cauty, 1991)

The space of all differentiable functions $f : I \to \mathbb{R}$ *is homeomorphic to the Hurewicz set* $\mathcal{H}$.

Π^1_1-ABSORBERS (COANALYTIC ABSORBERS)

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Π^1_1-ABSORBERS (COANALYTIC ABSORBERS)

Y is *strongly countable dimensional* if Y is a countable union of its compact, finite-dimensional subspaces.

X is *strongly infinite-dimensional* if there exists a sequence $(A_n, B_n)_n$ of closed disjoint subsets of X such that for each sequence $(C_n)_n$ of closed separators of X between A_n and B_n we have $\bigcap_n C_n \neq \emptyset$.

A space is *weakly infinite-dimensional* if it is not strongly infinite-dimensional.

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Π^1_1-absorbers (coanalytic absorbers)

$\mathcal{SCD}_n(X)$ – the family of all strongly countable-dimensional compacta of dimension $\geq n$ in X.

$\mathcal{W}_n(X)$ – the family of all weakly infinite-dimensional compacta of dimension $\geq n$ in X.

Theorem

Let X be a locally connected continuum such that each non-empty open subset of X contains a copy of the Hilbert cube.

- *$\mathcal{SCD}_n(X)$ and $\mathcal{W}_n(X)$ are coanalytic absorbers in 2^X for $n \geq 1$.*
- *$\mathcal{SCD}_n(X) \cap C(X)$ and $\mathcal{W}_n(X) \cap C(X)$ are coanalytic absorbers in $C(X)$ for $n \geq 2$.*

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Π_1^1-ABSORBERS (COANALYTIC ABSORBERS)

$\mathcal{SCD}_n(X)$ – the family of all strongly countable-dimensional compacta of dimension $\geq n$ in X.
$\mathcal{W}_n(X)$ – the family of all weakly infinite-dimensional compacta of dimension $\geq n$ in X.

THEOREM

Let X be a locally connected continuum such that each non-empty open subset of X contains a copy of the Hilbert cube.

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Π^1_1-absorbers (coanalytic absorbers)

X is *hereditarily locally connected* = each subcontinuum of X is locally connected.

$\mathcal{HLC}(X)$ – the family of all hereditarily locally connected subcontinua of X

X is *Suslinian* = each collection of pairwise disjoint nondegenerate subcontinua of X is countable.

$Susl(X)$ – the family of all Suslinian subcontinua of X

Theorem (Darji, Marcone, 2004)

If $n \geq 2$ then the families $\mathcal{HLC}(I^n)$ and $Susl(I^n)$ are coanalytic complete.

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Thank you!