Rojas– Sánchez & Tamariz-Mascarúa

Star-P and Weakly Star-P properties

Alejandro Darío Rojas-Sánchez

joint work with Ángel Tamariz-Mascarúa

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Spaces are considered Hausdorff and with at least two points. If X is a topological space and \mathcal{U} is a family of subsets of X, then the star of a subset $A \subset X$ with respect to \mathcal{U} is the set

$$st(A, U) = \bigcup \{U \in U : U \cap A \neq \emptyset\}$$

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$$st(A, U) = \bigcup \{U \in U : U \cap A \neq \emptyset\}$$

Definition

Let *P* be a topological property. A space *X* is said to be star-*P* if whenever \mathcal{U} is an open cover of *X*, there is a subspace $A \subset X$ with the property *P*, such that $st(A, \mathcal{U}) = X$. The set *A* will be called a star kernel of \mathcal{U} .

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Definition

The spread s(X) of a space X is defined as

$$s(X) = \sup \{ |A| : A \subset X \text{ is discrete} \}$$

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The extent e(X) of a space X is defined as

 $e(X) = \sup \{ |A| : A \subset X \text{ is closed and discrete} \}$

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In the following star-CS denote the star-(countable spread) property and star-CE denote the star-(countable extent) property.

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Some well-known implications:

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Some well-known implications:

• separable \Rightarrow star-countable

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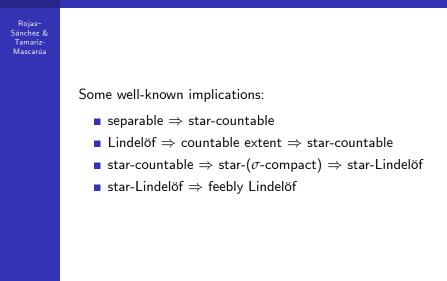
Some well-known implications:

- separable \Rightarrow star-countable
- Lindelöf \Rightarrow countable extent \Rightarrow star-countable



Some well-known implications:

- separable \Rightarrow star-countable
- Lindelöf \Rightarrow countable extent \Rightarrow star-countable
- star-countable \Rightarrow star-(σ -compact) \Rightarrow star-Lindelöf



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Definition

A topological space X is called feebly Lindelöf if every locally finite family of non-empty open sets in X is countable.

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Proposition

If X has an uncountable locally finite cellular family, then X is not a star-CE space.

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Definition

A topological space X is called feebly Lindelöf if every locally finite family of non-empty open sets in X is countable.

Proposition

If X has an uncountable locally finite cellular family, then X is not a star-CE space.

Corollary

If X is star-CE then X is feebly Lindelöf.

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Corollary

If X is star-CE then X is feebly Lindelöf.

star-Lindelöf \Rightarrow star-CE \Rightarrow feebly Lindelöf

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Example

If $S = \{\alpha + 1 : \alpha < \omega_1\}$ and $X = (\omega_1 \times \omega) \cup (S \times \{\omega\})$ is considered as a subspace of $\omega_1 \times (\omega + 1)$, then X is a Tychonoff star-CE space which is not star-Lindelöf.

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star-CE \Rightarrow star-Lindelöf

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Another implications:

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Another implications:

• Separable \Rightarrow CCC \Rightarrow star-CCC

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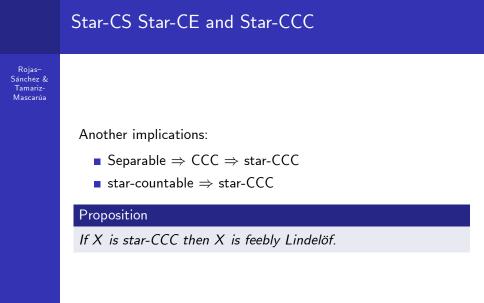
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Another implications:

• Separable \Rightarrow CCC \Rightarrow star-CCC

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• star-countable \Rightarrow star-CCC



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Example

Let \mathcal{A} be a MAD family in ω with $|\mathcal{A}| = 2^{\omega}$, and let $X = \Psi(\mathcal{A})$ be the psi-space related to \mathcal{A} . Let $Y = \alpha D$ be the one-point compactification of the discrete space D of size 2^{ω} . Then the space $X \times Y$ is a Tychonoff star-Lindelöf space which is not star-CCC.

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star-Lindelöf \Rightarrow star-CCC

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star-Lindelöf \Rightarrow star-CCC

Example

Let $\mathcal{F}[\mathbb{R}]$ be the hyperspace of all the non-empty finite subsets of \mathbb{R} endowed with the Pixley-Roy topology. Then $\mathcal{F}[\mathbb{R}]$ is a CCC Moore space wich is not star-CE.

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Let \mathcal{A} be a MAD family in ω with $|\mathcal{A}| = 2^{\omega}$, and let $X = \Psi(\mathcal{A})$ be the psi-space related to \mathcal{A} . Let $Y = \alpha D$ be the one-point compactification of the discrete space D of size 2^{ω} . Then the space $X \times Y$ is a Tychonoff star-Lindelöf space which is not star-CCC.

star-Lindelöf \Rightarrow star-CCC

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Let $\mathcal{F}[\mathbb{R}]$ be the hyperspace of all the non-empty finite subsets of \mathbb{R} endowed with the Pixley-Roy topology. Then $\mathcal{F}[\mathbb{R}]$ is a CCC Moore space wich is not star-CE.

star-CCC \Rightarrow star-CE

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More implications:

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More implications:

• countable spread \Rightarrow star-CS

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More implications:

• countable spread \Rightarrow star-CS

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• star-countable \Rightarrow star-CS

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More implications:

• countable spread \Rightarrow star-CS

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- star-countable \Rightarrow star-CS
- star-CS \Rightarrow star-CE

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More implications:

• countable spread \Rightarrow star-CS

- star-countable \Rightarrow star-CS
- star-CS \Rightarrow star-CE
- star-CS \Rightarrow star-CCC

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Theorem (Šapirovskii)

If $s(X) \leq \kappa$ then there is a dense subspace $Y \subset X$ with $hL(Y) \leq \kappa$.

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Theorem (Šapirovskii)

If $s(X) \leq \kappa$ then there is a dense subspace $Y \subset X$ with $hL(Y) \leq \kappa$.

Proposition

A space X is star-CS if and only if X is star-(hereditarily Lindelöf).

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Theorem (Šapirovskii)

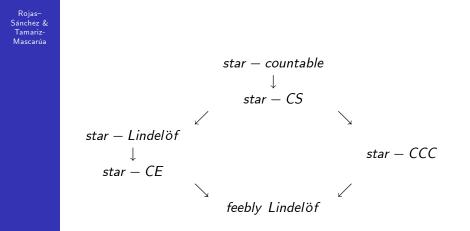
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 $star-CS \Rightarrow star-Lindelöf$



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Proposition

There is a Hausdorff star-CS space which is not star-countable.

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Proposition

There is a Hausdorff star-CS space which is not star-countable.

A measurable set $A \subset \mathbb{R}$ has density d at $x \in \mathbb{R}$ if

$$\lim_{h\to 0}\frac{m\left(A\cap\left[x-h,x+h\right]\right)}{2h}$$

exists and equals d. Denote $\phi(A) = \{x \in \mathbb{R} : d(x, A) = 1\}$ and let τ_d be the family of all measurable sets A such that $A \subset \phi(A)$. τ_d is a non-normal Tychonoff topology on \mathbb{R} which is stronger than the usual topology.

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Proposition

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exists and equals d. Denote $\phi(A) = \{x \in \mathbb{R} : d(x, A) = 1\}$ and let τ_d be the family of all measurable sets A such that $A \subset \phi(A)$. τ_d is a non-normal Tychonoff topology on \mathbb{R} which is stronger than the usual topology.

Proposition

[CH] The space (\mathbb{R}, τ_d) is a Tychonoff star-CS space wich is not star-countable.

Star-CS Star-CE and Star-CCC Subspaces

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> The star-CS, star-CE and star-CCC properties are not necessarily inherited by closed sets or closed G_δ-sets or zero sets.

Star-CS Star-CE and Star-CCC Subspaces

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- The star-CS, star-CE and star-CCC properties are not necessarily inherited by closed sets or closed G_δ-sets or zero sets.
- The star-CS, star-CE and star-CCC properties are not necessarily inherited by open sets or dense subspaces nor even by an open dense subspace.

Subspaces

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Proposition

The star-CS, star-CE and star-CCC properties are inherited over open F_{σ} -sets.

Subspaces

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Proposition

The star-CS, star-CE and star-CCC properties are inherited over open F_{σ} -sets.

Corollary

The star-CS, star-CE and star-CCC properties are inherited over clopen sets.

Subspaces

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Proposition

The star-CS, star-CE and star-CCC properties are inherited over open F_{σ} -sets.

Corollary

The star-CS, star-CE and star-CCC properties are inherited over clopen sets.

Corollary

The star-CS, star-CE and star-CCC properties are inherited over cozero sets.

Star-CS Star-CE and Star-CCC Subspaces

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Example

Let \mathcal{A} be a MAD family in ω , with $|\mathcal{A}| = 2^{\omega}$, and let αD be the one-point compactification of the discrete space D of size 2^{ω} . The space $X_1 = \mathcal{A} \cup (\omega \times \alpha D)$, where $\omega \times \alpha D$ is an open subspace of X_1 and an open neighborhood for $A \in \mathcal{A}$ takes the form

$$O_{F,U}(A) = \{A\} \cup ((A \setminus F) \times U)$$

where $F \subset A$ is finite and $U \subset \alpha D$ is open, is a non star-CCC Tychonoff space.

Subspaces

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Proposition

There is a Tychonoff star-countable space with a regular closed set non star-CCC.

Subspaces

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Proposition

There is a Tychonoff star-countable space with a regular closed set non star-CCC.

Proof.

Considering the non star-CCC space $X_1 = \mathcal{A} \cup (\omega \times \alpha D)$ and the psi-space $X_2 = \Psi(\mathcal{A})$ construct a quotient space Xresulting from identify the subspace \mathcal{A} of X_1 with the subspace \mathcal{A} of X_2 . The space X is a star-countable space and has an homeomorphic copy of X_1 as a regular closed subset.

Subspaces

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Proposition

There is a Tychonoff star-countable space with a regular closed set non star-CE.

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Subspaces

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

There is a Tychonoff star-countable space with a regular closed set non star-CE.

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Proposition

There is a Lindelöf space with a regular open set which is neither a star-CE space nor star-CCC space.

Finite products

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Proposition

If X is star-CE and K is a compact space then $X \times K$ is star-CE.

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Proposition

If X is star-CE and K is a compact space then $X \times K$ is star-CE.

Example

The space $X = \Psi(A) \times \alpha D$ is not star-CCC despite of being the product of a separable space and a compact space.

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

If X is star-CE and K is a compact space then $X \times K$ is star-CE.

Example

The space $X = \Psi(A) \times \alpha D$ is not star-CCC despite of being the product of a separable space and a compact space.

Proposition

If X is star-CCC (star-CS) and K is a compact separable space then $X \times K$ is a star-CCC (star-CS) space.

Star-CS Star-CE and Star-CCC Finite products

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Proposition

If $e(X) > \omega$ and $c(Y) > \omega$ then $X \times Y$ is not a star-CCC space.

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Proposition

If $e(X) > \omega$ and $c(Y) > \omega$ then $X \times Y$ is not a star-CCC space.

Problem

Is the product of a star-CCC (star-CS) space and a compact CCC space, star-CCC (star-CS)?

Star-CS Star-CE and Star-CCC Finite products

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

There exist a countably compact space X and a Lindelöf space Y such that $X \times Y$ is neither star-CE nor star-CCC.

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Proposition

There exist a countably compact space X and a Lindelöf space Y such that $X \times Y$ is neither star-CE nor star-CCC.

Proposition

There exist two countably compact spaces X and Y such that $X \times Y$ is neither star-CE nor star-CCC.

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There exist two countably compact spaces X and Y such that $X \times Y$ is neither star-CE nor star-CCC.

Proposition

There exist two Lindelöf spaces X and Y such that $X \times Y$ is neither star-CE nor star-CCC.

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A subset $A \subset \mathbb{R}$ is *totally imperfect* if A does not contain a copy of the Cantor set.

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A subset $A \subset \mathbb{R}$ is *totally imperfect* if A does not contain a copy of the Cantor set.

Theorem (Bernstein)

There exists $A \subset \mathbb{R}$ such that $|A| = |\mathbb{R} \setminus A| = 2^{\omega}$ and either A or $\mathbb{R} \setminus A$ are totally imperfect sets.

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A subset $A \subset \mathbb{R}$ is *totally imperfect* if A does not contain a copy of the Cantor set.

Theorem (Bernstein)

There exists $A \subset \mathbb{R}$ such that $|A| = |\mathbb{R} \setminus A| = 2^{\omega}$ and either A or $\mathbb{R} \setminus A$ are totally imperfect sets.

Example

Define a finner topology than the Euclidian on $Y = A \cup (\mathbb{R} \setminus A)$ by isolating each point in $\mathbb{R} \setminus A$. Y endowed with this topology is a Lindelöf space. Take X = A with the subspace topology and call $M = X \times Y$.

Finite products

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Proposition

The space M has the following properties:

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Star-CS Star-CE and Star-CCC Finite products

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Proposition

The space M has the following properties:

M is the product of a second countable space with a Lindelöf space.

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2 *M* is not star-CCC.

Star-CS Star-CE and Star-CCC Finite products

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Proposition

The space M has the following properties:

 M is the product of a second countable space with a Lindelöf space.

- 2 *M* is not star-CCC.
- 3 M is star-Lindelöf.

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

The space M has the following properties:

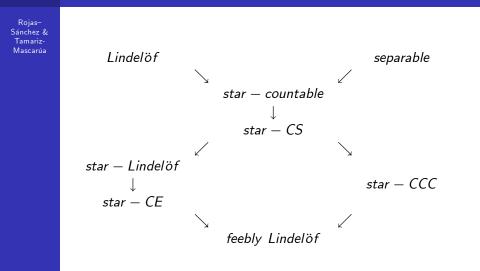
- M is the product of a second countable space with a Lindelöf space.
- 2 M is not star-CCC.

3 *M* is star-Lindelöf.

Problem

Is the product of a star-CE and second countable space, star-CE?

Relations with other covering properties



Relations with other covering properties

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Definition

A space X is paralindelöf (σ -paralindelöf) if every open cover of X has a locally countable (σ -locally countable) open refinement.

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Relations with other covering properties

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A space X is paralindelöf (σ -paralindelöf) if every open cover of X has a locally countable (σ -locally countable) open refinement.

Proposition (Blair)

Every paralindelöf feebly Lindelöf space is Lindelöf.

Relations with other covering properties

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Problem

Does every σ -paralindelöf feebly Lindelöf space is Lindelöf?

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Proposition (Hiremath)

If X is σ -paralindelöf then X is Lindelöf if and only if X is star-Lindelöf.

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Proposition (Hiremath)

If X is σ -paralindelöf then X is Lindelöf if and only if X is star-Lindelöf.

Proposition

If X is σ -paralindelöf then X is Lindelöf if and only if X is star-CCC.

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

If X is σ -paralindelöf and contains a dense subspace with countable extent then X is Lindelöf.

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Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

If X is σ -paralindelöf and contains a dense subspace with countable extent then X is Lindelöf.

Corollary

If X is σ -paralindelöf then X is Lindelöf if and only if X is star-CE.

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

If X is σ -paralindelöf and contains a dense subspace with countable extent then X is Lindelöf.

Corollary

If X is σ -paralindelöf then X is Lindelöf if and only if X is star-CE.

Proof.

If \mathcal{U} is an open cover and $M \subset X$ is a star kernel of \mathcal{U} with $e(M) \leq \omega$, then $cl_X M$ is Lindelöf and is a star kernel of \mathcal{U} . Thus X is Lindelöf.

Relations with other covering properties

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Corollary

Let X be a σ -paralindelöf space. The following are equivalent:

Relations with other covering properties

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Corollary

Let X be a σ -paralindelöf space. The following are equivalent:

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i) X is Lindelöf

Relations with other covering properties

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Corollary

Let X be a σ -paralindelöf space. The following are equivalent:

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i) X is Lindelöfii) X is star-CS

Relations with other covering properties

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Corollary

Let X be a σ -paralindelöf space. The following are equivalent:

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i) X is Lindelöf
ii) X is star-CS
iii) X is star-CE

Relations with other covering properties

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Corollary

Let X be a σ -paralindelöf space. The following are equivalent:

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i) X is Lindelöf
ii) X is star-CS
iii) X is star-CE
iv) X is star-CCC

Relations with other covering properties

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Theorem (Alas, Junqueira, van Mill, Tkachuk, Wilson, 2011)

If X is a Moore space, then X is separable if and only if X is star-Lindelöf.

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Theorem (Alas, Junqueira, van Mill, Tkachuk, Wilson, 2011)

If X is a Moore space, then X is separable if and only if X is star-Lindelöf.

Corollary

If X is a Moore space, then X is separable if and only if X is star-CE.

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Theorem (Alas, Junqueira, van Mill, Tkachuk, Wilson, 2011)

If X is a Moore space, then X is separable if and only if X is star-Lindelöf.

Corollary

If X is a Moore space, then X is separable if and only if X is star-CE.

Proposition

If X is a semistratifiable space, then X is star-countable if and only if X is star-CE.

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Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

The hyperspace $\mathcal{F}\left[\mathbb{R}\right]$ with the Pixley-Roy topology is a Moore CCC space which is not star-CE.

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

> The hyperspace $\mathcal{F}[\mathbb{R}]$ with the Pixley-Roy topology is a Moore CCC space which is not star-CE. When is the hyperspace $\mathcal{F}[X]$ a star-CE space?

Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

For any space X, $\mathcal{F}[X]$ has the following properties:

Relations with other covering properties

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> For any space X, $\mathcal{F}[X]$ has the following properties: i) $\mathcal{F}[X]$ is a hereditarily metacompact space.

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Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

> For any space X, $\mathcal{F}[X]$ has the following properties: i) $\mathcal{F}[X]$ is a hereditarily metacompact space. ii) $\mathcal{F}[X]$ is a Moore space if and only if $\chi(X) \leq \omega$.

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Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

 $\mathcal{F}[X]$ is star-countable if and only if $|X| \leq \omega$.

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Relations with other covering properties

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

$$\mathcal{F}[X]$$
 is star-countable if and only if $|X| \leq \omega$.

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If $\mathcal{F}[X]$ is star-CE then X is hereditarily Lindelöf.

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Proposition

$$\mathcal{F}[X]$$
 is star-countable if and only if $|X| \leq \omega$.

Proposition

If
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 is star-CE then X is hereditarily Lindelöf.

Corollary

$$\mathcal{F}[X]$$
 is star-CE if and only if $|X| \leq \omega$.

Rojas– Sánchez & Tamariz-Mascarúa

Definition

A space X is weakly Lindelöf if for every open cover \mathcal{U} of X there is a countable subset $\mathcal{U}_0 \subset \mathcal{U}$ such that $\bigcup \mathcal{U}_0$ is dense in X.

Rojas– Sánchez & Tamariz-Mascarúa

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Proposition (Sakai)

If $\mathcal{F}[X]$ is weakly Lindelöf, then X is hereditarily Lindelöf. In addition, if $t(X) \leq \omega$ then X is hereditarily separable.

Rojas– Sánchez & Tamariz-Mascarúa

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If $\mathcal{F}[X]$ is weakly Lindelöf, then X is hereditarily Lindelöf. In addition, if $t(X) \leq \omega$ then X is hereditarily separable.

Definition

Let *P* be a topological property. A space *X* is said to be weakly star-*P* if whenever \mathcal{U} is an open cover of *X*, there is a subspace $A \subset X$ with the property *P*, such that $st(A, \mathcal{U})$ is dense in *X*.

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weakly Lindelöf \Rightarrow weakly star-countable

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weakly Lindelöf \Rightarrow weakly star-countable

Proposition

If X is weakly star-countable and metalindelöf then X is weakly Lindelöf.

Rojas– Sánchez & Tamariz-Mascarúa

Proposition

If $\mathcal{F}[X]$ is weakly star-CE, then $\mathcal{F}[X]$ is weakly star-countable.

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Rojas– Sánchez & Tamariz-Mascarúa

Proposition

If $\mathcal{F}[X]$ is weakly star-CE, then $\mathcal{F}[X]$ is weakly star-countable.

Corollary

If $\mathcal{F}[X]$ is weakly star-CE, then X is hereditarily Lindelöf. In addition, if $t(X) \leq \omega$ then X is hereditarily separable.

Star-P and weakly star-P properties

Rojas– Sánchez & Tamariz-Mascarúa

Thank you

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