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for membership in $\mathcal{PP}$.

His condition involves the topological game $\mathcal{G}(X, \mathcal{DC})$,
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which are discrete unions of compact sets.

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The implication B$\Rightarrow$C is a consequence of the following result.

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We have $\tau \subset \pi$, and hence Z is Hausdorff.

To see that Z is paracompact, note that X^* is paracompact in Z, and use the result of Wicke and Worrell that a locally finite open family of a subspace can be extended to a locally-finite-in-itself open family of the closure of the subspace.

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Hence $\big(U^{-1}K\big) \cap \big(W \setminus G\big) = \emptyset$, i.e., $\big(U^{-1}K \setminus G\big) \cap W = \emptyset$.

This shows that $p \notin \mathrm{Cl}_{\beta X}\big(U^{-1}K \setminus G\big)$. $\square$

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Proposition *Assume that X is productively paracompact and paracompact at infinity. Then the subspace $X \setminus LC(X)$ has a strongly co-compact neighbornet.*

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Proof. Consider the nowhere locally compact space $X \times \mathbb{Q}$. $\square$

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Proposition *Let X be productively paracompact, suborderable and of point-countable type. Then X has a co-compact neighbornet.*

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By a result of Kofner, U^3 contains a transitive neighbornet V.

With nowhere local compactness we get a stronger conclusion.

Proposition *A nowhere locally compact productively paracompact suborderable space X of point-countable type has a transitive strongly co-compact neighbornet.*

Proof. *By results mentioned earlier, X has a strongly co-compact neighbornet U. Note that each neighbornet U^n is strongly co-compact. By a result of Kofner, U^3 contains a transitive neighbornet V. The neighbornet V is strongly co-compact.* □

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E. *X is σ-locally compact.*