

# On stability and weight of Lindelöf $P$ -groups

*Mikhail Tkachenko*<sup>1</sup>

mich@xanum.uam.mx

If every  $G_\delta$ -set in a space  $X$  is open, we say that it is a  $P$ -space. Similarly, a topological group is a  $P$ -group if it is a  $P$ -space. The class of Lindelöf  $P$ -groups, which is similar to the class of compact topological groups in terms of permanence properties, is the focus of our presentation.

First, we consider the question of whether every Lindelöf  $P$ -group is  $\tau$ -stable, for a given infinite cardinal  $\tau$ . It is well known that the answer is affirmative for  $\tau = \aleph_0$  and  $\tau = \aleph_1$ . We extend this conclusion to a proper class of cardinals  $\tau$ , including those satisfying the equality  $\tau^\omega = \tau$ . We deduce, for example, that every Lindelöf  $P$ -group is  $\aleph_n$ -stable, for each  $n \in \omega$ .

Second, we look at the problem of estimating a gap between the weight and  $i$ -weight of a Lindelöf  $P$ -group  $G$ . If the cardinal  $\tau = iw(G)$  is either  $\aleph_n$  for some  $n \in \omega$  or fulfills  $\tau^\omega = \tau$ , it is not difficult to demonstrate that the two cardinal functions coincide. In general, however, a gap can be quite big. According to our best knowledge, if  $iw(G) = \aleph_\omega$ , then  $w(G) < \aleph_{\omega_4}$  and  $w(G) \leq (\tau_\omega)^\omega$ .

Our entire analysis is based on the family  $[\tau]^\omega$  of countable subsets of an uncountable cardinal  $\tau$  partially ordered by inclusion.

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