Countable tightness and the Grothendieck property in $C_p(X)$; other recent results

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A famous theorem of Grothendieck widely used in Analysis asserts that if X is a compact space and A is a subspace of $C_p(X)$, then the closure of A is compact if and only if every infinite subset of A has a limit point in $C_p(X)$. Casazza and Iovino used Grothendieck's Theorem to prove the undefinability of pathological Banach spaces in compact logics. C. Hamel and I extended their work to a large class of logics with type spaces satisfying Grothendieck's Theorem. Recently we have extended these results to logics with countably tight type spaces. Arhangel'skiĭ extensively studied generalizations of Grothendieck's Theorem in a 1998 paper and raised several problems. We answer many of these, showing them undecidable in ZFC, for example whether Lindelöf countably tight spaces satisfy the conclusion of Grothendieck's Theorem. Our affirmative answer (true, for example, under PFA) is a dramatic improvement of the previous result that required, under MA_{ω_1}, in addition that all finite powers of X be Lindelöf. We also provide a counterexample under $V = L$ to the MA_{ω_1} conclusion. The proofs require too much C_p-theory to do in a short talk; instead we mention other unpublished recent work that may be of interest. M. Morley proved in 1972 that the number of countable non-isomorphic models of a first-order theory is either countable, of size continuum, or of size $\aleph_1$. With C. Hamel, C. Eagle, and S. Müller, we prove that the analogous claim for second-order theories is undecidable. For those who do not attend my Toposym talk, let me mention that with Ivan Ongay Valverde, I have investigated various topological generalizations of descriptive set theory, most notably the class of upper semi-continuous compact-valued images of σ-projective sets of reals. These generalize the K-analytic spaces and a number of results about those spaces (e.g. regarding Selection Principles) do generalize, assuming large cardinals which imply determinacy axioms.

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