Pinning Down versus Density

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The *pinning down number* $pd(X)$ of a topological space X is the smallest cardinal κ such that for any neighborhood assignment $U : X \to \tau_X$ there is a set $A \in [X]^\kappa$ with $A \cap U(x) \neq \emptyset$ for all $x \in X$. Clearly, $c(X) \leq pd(X) \leq d(X)$.

The following statements are equivalent: (a) $2^\kappa < \kappa^{+\omega}$ for each cardinal κ; (b) $d(X) = pd(X)$ for each Hausdorff space X; (c) $d(X) = pd(X)$ for each 0-dimensional Hausdorff space X; (d) $d(X) = pd(X)$ for each Abelian topological group X; (e) $d(X) = pd(X)$ for each connected, locally connected, homogeneous, regular space X.

Let (f) be the following statement: $d(X) = pd(X)$ for each connected, Tychonoff space X. We proved that (f) is strictly weaker than (a)-(e) above, but the failure of (f) is still consistent.

We show that the following three statements are *equiconsistent*:

1. There is a singular cardinal λ with $pp(\lambda) > \lambda^+$, i.e. Shelah's Strong Hypothesis fails;
2. there is a 0-dimensional Hausdorff space X such that $|X| = \Delta(X)$ is a regular cardinal and $pd(X) < d(X)$;
3. there is a topological space X such that $|X| = \Delta(X)$ is a regular cardinal and $pd(X) < d(X)$.

We discuss cardinal inequalities involving $pd(X)$.

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