Structure of groups of order-preserving homeomorphisms of product spaces with lexicographic order and their compactifications

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Let X and Y be linearly ordered spaces, $H_+(X)$ and $H_+(Y)$ are their groups of order-preserving homeomorphisms. Let $\mathrm{Hom}_+^\partial(X)$ be the group $H_+(X)$ in the permutation topology and $\mathrm{Hom}_+(Y)$ be the group $H_+(Y)$ in the topology of pointwise convergence.

Theorem. *If each order preserving homeomorphism of the product $X \times Y$ with lexicographic order maps every fiber $\{x\} \times Y$ onto fiber $\{x'\} \times Y$, then the group of order-preserving homeomorphisms of the product $X \times Y$ with lexicographic order in the topology of pointwise convergence is topologically isomorphic to the semi-direct topological product $G \ltimes T$ where G is the power $\mathrm{Hom}_+(Y)^X$ in Tikhonoff topology and T is $\mathrm{Hom}_+^\partial(X)$.*

Corollary. *The group of order-preserving homeomorphisms in the topology of pointwise convergence of*

(1) *the lexicographically ordered square* $\mathbf{K}$ *is topologically isomorphic to* $\mathrm{Hom}_+([0,1])^{[0,1]} \ltimes \mathrm{Hom}_+^\partial([0,1])$*;*

(2) *the lexicographically ordered square of the real line* $\mathbf{R}$ *is topologically isomorphic to* $\mathrm{Hom}_+(\mathbb{R})^{\mathbb{R}} \ltimes \mathrm{Hom}_+^\partial(\mathbb{R})$*;*

(3) *the "Double Arrow" Alexandroff space* $\mathbf{D}$ *is topologically isomorphic to* $\mathrm{Hom}_+^\partial([0,1])$.