Čech and Katětov covering dimensions and more or less related questions concerning F-groups

Olga Sipacheva

ovsipa@gmail.com

There are two covering dimensions, dim in the sense of Čech ($\dim X \le n$ if any finite open cover of X has a finite open refinement of order $\le n$) and $\dim_0$ in the sense of Katětov ($\dim_0 X \le n$ if any finite cozero cover of X has a finite cozero refinement of order $\le n$). It is proved that the covering dimension of the Sorgenfrey plane $S \times S$ is infinite, while, as is well known, $\dim_0 S^\kappa = 0$ for any cardinal κ. Examples of topological groups with similar properties are constructed, including a separable precompact Boolean group G with linear topology (generated by open subgroups) with $\dim_0 G^\kappa = 0$ and $\dim G = \infty$.

The open problem of the existence in ZFC of topological groups whose underlying space is an F-space (i.e., a space in which any two disjoint cozero sets are functionally separated) not being P-spaces is touched on. It is proved that the existence of an Abelian F-group G such that $\dim_0 G < \infty$ and $\psi(G) \le \omega$ is equivalent to the existence of a Boolean group with the same properties and that the existence of an Abelian F'-group (or of an extremally disconnected group) G with linear topology which is not a P-space implies the existence of a group of cardinality $\le 2^\omega$ with the same properties.

Question. Is it true that $\dim_0 X \le \dim X$ for any completely regular space X?