Continuity in right semitopological groups and semineighbourhoods of the diagonal

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Let X be a space and $P \subset X \times X$. We call P a *semineighbourhood of the diagonal* if $x \in \operatorname{Int} P(x)$ for all $x \in X$, where $P(x) = \{y \in G : (x, y) \in P\}$. A space X is called Δ-*nonmeager* (Δ_h-*;* Δ_s-*nonmeager*) if for any semineighbourhood of the diagonal P there exists a non-empty open $W \subset X$ such that that the condition (Δ) $W^2 \subset \overline{P}$ (respectively, (Δ_h) the set $\{x \in W : (x, y) \in P\}$ is dense in W for all $y \in W$; (Δ_h) the set $\{x \in W : (x, y) \in P$ for all $y \in W\}$ is dense in W) holds. A mapping $f : X \to Y$ is called *feebly continuous* if $\operatorname{Int} f^{-1}(U) \neq \emptyset$ for any open $U \subset Y$ such that $U \cap f(X) \neq \emptyset$.

Theorem. *[1] Let G be a regular right topological group such that every left shift $x \mapsto gx$ is feebly continuous. Let us assume that one of the following conditions is met: G is Δ-nonmeager and multiplication in G is feebly continuous; G is Δ_h-nonmeager and the operation $x \mapsto x^{-1}$ is continuous; G is Δ_s-nonmeager. Then G is a topological group.*

[1] E. Reznichenko, *Continuity in right semitopological groups*, 2022. Preprint, arXiv:2205.06316.