Maximal Homogeneous Spaces

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We say that a space X is *maximal homogeneous* if X is a maximal homogeneous subspace of βX containing X. For any homogeneous space X, there exists a unique maximal homogeneous space $H(X) \subset \beta X$ for which $X \subset H(X) \subset \beta X$. For example, any first countable homogeneous space is maximal homogeneous. Let p be a free ultrafilter on ω. We say that a space X *totally countably p-compact* if, for any infinite $M \subset X$, there exists an infinite $L \subset M$ such that any sequence $(x_n)_{n\in\omega} \subset L$ ($x_i \neq x_j$ for $i \neq j$) has a p-limit in X. Any totally countably compact space (in particular, any sequentially compact space) is totally countably p-compact for any $p \in \omega^* = \beta\omega \setminus \omega$. Clearly, any totally countably p-compact space is contable compact.

Theorem *If $p \in \omega^*$ and X is totally countably p-compact space, then X^ω is totally countably p-compact and, hence, countably compact.*

Theorem *Let X be a maximal homogeneous extremally disconnected space. If X contains a nonclosed dicrete sequence of points, then X is totally countably p-compact for some $p \in \omega^*$.*

Note that all known examples of homogeneous extremally disconnected countably compact spaces are maximally homogeneous.

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