

On the problem of amenability of groups of maps

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Given a compact space X and a compact group K , is the group $C(X, K)$ of all continuous maps with the topology of uniform convergence amenable? (Recall that a topological group is amenable if every continuous action of the group on a compact space admits an invariant regular probability measure.)

The only case where the answer is quite trivially “yes” is where $\dim X = 0$. Already in the one-dimensional case of $X = [0, 1]$ or S^1 , an affirmative answer, due to Marie-Paule Malliavin and Paul Malliavin (1992), requires using a subtle machinery of Wiener measures and going to maps of a higher smoothness class. Beyond those two settings, nothing seems to be known in the continuous case.

Actually, one needs to look at the higher smoothness degree maps also in order to understand the motivation of the question, which comes from mathematical physics. The analysis of the problem shows that perhaps the property needed is not exactly amenability, but one of its variants which is equivalent to amenability in the locally compact case: skew-amenability.

We will discuss the motivation, the background, the techniques, known results and open questions, reviewing, in addition to the above mentioned work, some recent results by the speaker and also by Kate Juschenko and F. Martin Schneider.

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