

# ZFC solution to 9 problems of Tkachuk on functional countability

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A Tychonoff space is called *functionally countable* if every real-valued continuous function has countable range. A compact space is functionally compact iff it is scattered. At this year's STDC conference, Vladimir Tkachuk posed 14 problems on functional countability. A ZFC solution is given for 9 of these problems, of which the most demanding is Question 8: If  $X$  is a compact, Fréchet–Urysohn space such that  $X^2 \setminus \Delta_X$  is functionally countable, must  $X$  be separable? The counterexample has  $2^\omega$  isolated points and is the one-point compactification of a finer topology on  $(0, 1)$  such that the non-isolated points form a closed discrete subspace.