A structured construction of locally compact spaces by induction

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An archetypal example of constructing spaces by induction was given by Ostaszewski in constructing his S-space under the assumption of $\diamondsuit$. This talk introduces a much more structured type of induction in showing:

Theorem *Every stationary, co-stationary subset E of ω_1 has a locally compact, normal, quasi-perfect preimage X of cardinality $\mathfrak{b}$.*

Assuming wo l o g that the successor ordinals are the isolated points of E, let E_0 be the set of points of E which are not in the closure of $\omega_1 \setminus E$. The underlying set for X is $E_0 \cup [(E \setminus E_0) \times \mathfrak{b}]$.

The neighborhoods of $(\alpha, \xi) \in [(E \setminus E_0) \times \mathfrak{b}]$ hit an array of clopen sets defined in earlier stages in several precise ways with the use of a family of increasing functions, $f_\eta : \omega \to \omega$ $(\eta < \mathfrak{b})$ that are well-ordered and unbounded in the eventual domination order $<^*$.

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