On uniform real complete spaces

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A uniform space (X, U) is called pre-real compact if its uniformity U is generated by all uniform continuous functions [1] [2].

Theorem. *For each uniform space (X, U) there is exactly one (up to an uniform homeomorphism) uniformly real complete space $(\theta_U X, \theta_U)$ with the following properties:*

(1) *There is an uniformly homeomorphic embedding $i : (X, U_F) \to (\theta_U X, \theta_U)$, for which $(\theta_U X, \theta_U)$ is the completion of the uniform space (X, U_F), where U_F is the maximal functional uniformity contained in U.*

(2) *For any continuous function $f : (X, U) \to (R, E_R)$, there is an uniformly continuous function $\tilde{f} : (\theta_U X, \theta_U) \to (R, E_R)$ such that $\tilde{f} \circ i = f$.*

Moreover, the spaces $(\theta_U X, \theta_U)$ also satisfy the condition:

(3) *For each uniformly continuous mapping $f : (X, U) \to (Y, \mathfrak{M})$ of the uniform space (X, U) into an arbitrary uniformly real complete space $(Y, \mathfrak{M})$, there is an uniform mapping $\tilde{f} : (\theta_U X, \theta_U) \to (Y, \mathfrak{M})$ such that $\tilde{f} \circ i = f$.*

[1] R. Engelking, *General topology*, Moscow, 1986 (in Russian).

[2] A. A. Borubaev, *Uniform topology and its applications*, Bishkek, 2021.