Some topological properties of the N_τ^φ-nucleus of a space X

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Let X be a T_1-space, φ be a cardinal-valued function, and τ be a cardinal number.

The N_τ^φ-nucleus of the space X is the space

$$N_\tau^\varphi X = \{\mathfrak{M} \in NX: \text{ there exists } F \in \mathfrak{M} \text{ such that } \varphi(F) \leq \tau\},$$

where, the set NX of all complete linked systems (CLS) of a space X.

Assume that $\mathfrak{M} \in N_\tau^\varphi$-basement of the CLS $\mathfrak{M}$ is the family

$$\mathfrak{F}_\tau^\varphi(\mathfrak{M}) = \{F \in \mathfrak{M}: \ \varphi(F) \leq \tau\}.$$

A topological space X is said to be N_τ^φ-nuclear if $N_\tau^\varphi X = NX$.

As φ, we take a density function d. Let $\tau = \omega$.

Theorem. *For every infinite compact space X the following conditions are equivalent:*

(1) *The space X satisfies the second axiom of countability;*

(2) *The space $N_\omega^d X$ satisfies the second axiom of countability;*

(3) *The space NX satisfies the second axiom of countability.*

Theorem. *For every infinite compact space X the following conditions are equivalent:*

(1) *The space X has the Souslin property;*

(2) *The space $N_\omega^d X$ has the Souslin property;*

(3) *The space NX has the Souslin property.*