Minimal non σ-scattered linear orders

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A linear order is *scattered* if it does not contain a copy of the rational line and *σ-scattered* if it is a countable union of scattered suborders. In 1971, Laver proved that the class of σ-scattered linear orders is *well quasi-ordered*: if L_i $(i < \infty)$ is a sequence of σ-scattered linear orders, then there is an $i < j$ such that L_i embeds into L_j. At the time, Laver speculated whether his result could be extended in ZFC to a broader hereditary class of linear orders (Baumgartner had shown around the same time that PFA implied that Laver's result could be extended to a broader class of linear orders). An equivalent form of this question can be stated as follows: is there a ZFC example of a linear orders which is minimal with respect to being non σ-scattered? We have proved that this is not the case. We have also shown that PFA^+ can be used to give a rough classification of the non σ-scattered linear orders: every non σ-scattered linear order contains either an Aronszajn type, a real type, or a ladder system indexed by a stationary subset of ω_1, equipped with either the lexicographic or reverse lexicographic order.

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