The Samuel Realcompactification

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In this talk we will introduce a realcompactification for any metric space (X, d), defined as the smallest realcompactification of X such that every real-valued uniformly continuous function can be continuously extended to it. It is called the *Samuel realcompactification* according to the well known Samuel compactification associated to the family of all the bounded real-valued uniformly continuous functions. Similarly, the Lipschitz realcompactification of (X, d) is defined as the smallest realcompactification of X such that every real-valued Lipschitz function can be continuously extended to it.

We will start showing how the Samuel realcompactification of (X, d) can be described in terms of the Lipschitz realcompactification of (X, ρ_λ) for a certain family of metrics $\{\rho_\lambda\}_\lambda$ all of them uniformly equivalent to d. This description will allow to deduce when the Samuel realcompactification of (X, d) is equivalent to the Lipschitz realcompactification of (X, ρ) for some metric ρ uniformly equivalent to d. This is in fact equivalent to a problem studied by J. Hejcman in the setting of metrizable bornologies. Next, we will give a Katetov-Shirota type theorem asserting that a metric space (X, d) is Samuel realcompact if and only if X satisfies a strong property of completeness, called Bourbaki-completeness (defined by the authors), and every uniformly discrete closed subspace of X has non-measurable cardinal. In particular, this result gives an answer, in the frame of metric spaces, to a question posed by Hušek and Pulgarín.

[1] Both authors were partially supported by the MINECO Project MTM2012-34341 (Spain).