On completely Baire spaces

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A space X is called *weakly Preiss–Simon* at a point $x \in X$ if there is a sequence $\langle U_n : n \in \omega \rangle$ of nonempty open subsets of X that converges to x. Denote by $wPS(X)$ the set of points $x \in X$ at which X is weakly Preiss–Simon. A space X is called *weakly Preiss–Simon* space if the set $wPS(X)$ is dense in X. A space X is called *hereditary weakly Preiss–Simon* (briefly, a hwPS-space) if every closed subset of X is a weakly Preiss–Simon space. Clearly, any first countable space is a hwPS-space.

The following theorem generalizes the classical Hurewicz theorem about closed embedding of the rationals into metrizable spaces.

Theorem. *Let X be a weakly Preiss–Simon regular space of the first category. Then X contains a closed copy of the space of rational numbers.*

The van Douwen theorem can be strengthened by the following

Theorem. *A regular hwPS-space X is completely Baire if and only if the space of rational numbers does not embed as a closed subspace into X.*

Preservation of complete Baireness is described by the following

Theorem. *Let $f : X \to Y$ be a continuous mapping of a completely Baire space X onto a regular hwPS-space Y such that the image of every open subset of X is a resolvable set in Y. Then Y is completely Baire.*