Resolvable-measurable mappings of metrizable spaces

Sergey Medvedev

medvedevsv@susu.ru

In an old question Lusin asked when a Borel mapping $f : X \to Y$ can be decomposed into continuous mappings $f_n : X_n \to Y$, where $\{X_n : n \in \omega\}$ is a cover of X and the restriction $f|X_n = f_n$. The first affirmative answer was given by J.E. Jayne and C.A. Rogers. Recall that a mapping $f : X \to Y$ is *piecewise continuous* if X can be covered by a sequence $X_0, X_1, \ldots$ of closed sets such that the restriction $f|X_n$ is continuous for every $n \in \omega$.

Theorem *Jayne–Rogers, 1982 If X is an absolutely Souslin-$\mathcal{F}$ metrizable space and Y is a metrizable space, then $f : X \to Y$ is $\mathbf{\Delta}^0_2$-measurable if and only if it is piecewise continuous.*

This theorem was generalized a number of different ways. We give a similar statement for a metrizable completely Baire space X. Some related results will be discussed.

A mapping $f : X \to Y$ is said to be *resolvable-measurable* if $f^{-1}(U)$ is a resolvable subset of X for every open set $U \subset Y$.

Theorem *Let $f : X \to Y$ be a mapping of a metrizable completely Baire space X to a regular space Y. Then f is resolvable-measurable if and only if it is piecewise continuous.*

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