Turning ternary relations into antisymmetric betweenness relations

Jorge Bruno, Aisling McCluskey, Paul Szeptycki*

brujo.email@gmail.com,
aisling.mccluskey@nuigalway.ie,
szeptyck@yorku.ca

Let $\mathcal{R}$ be a family of nonempty subsets of a set X such that

1. all singleton subsets of X are in $\mathcal{R}$, and
2. for any a, b in X, there is $R \in \mathcal{R}$ with $a, b \in R$.

A ternary relation then arises naturally on X from such a family by writing $[a, b, c]$ (and saying *b is between a and c*) if and only if $b \in R$ for each $R \in \mathcal{R}$ with $a, c \in R$. This primitive notion of betweenness was introduced by Bankston in 2013. He showed in particular that such relations, called R-relations, are first-order axiomatizable.

An R-relation is said to be antisymmetric if $[a, b, c]$ and $[a, c, b]$ together imply $b = c$. We construct the antisymmetric closure of an R-relation and expose it as a reflector between complete lattices and their distributive counterparts.

TOPOSYM
Prague 2016