# The Menger property is $l$-invariant

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For a Tychonoff space $X$, by $C_p(X)$ we denote the space of all continuous real-valued functions on $X$ endowed with the topology of pointwise convergence. Recall that a space $X$ has the *Menger property* if for every sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of open covers of $X$, there is a sequence $(\mathcal{V}_n)_{n\in\mathbb{N}}$ such that for every $n$, $\mathcal{V}_n$ is a finite subfamily of $\mathcal{U}_n$ and the family $\bigcup_{n\in\mathbb{N}}\mathcal{V}_n$ covers $X$.

An old question of A.V. Arhangel'skii asks if the Menger property of $X$ is preserved by homeomorphisms of the space $C_p(X)$. A similar question can also be asked for linear homeomorphisms of $C_p(X)$-spaces. In 2020 M. Sakai gave the affirmative answer in the linear case under an additional assumption on $X$. We show that the answer in the linear case is affirmative in the full generality. Our method can also be applied to prove analogous theorems for other, related covering type properties.