Arbitrarily large countably compact free Abelian groups

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A result of A. Tomita shows that a nontrivial free Abelian group does not admit a group topology whose countable power is countably compact.

In our work, we show that if there are $\mathfrak{c}$ incomparable selective ultrafilters, then, for every infinite cardinal κ such that $\kappa^\omega = \kappa$, there exists a group topology on the free Abelian group of cardinality κ without nontrivial convergent sequences and such that every finite power is countably compact.

This answers a question of Dikranjan and Shakhmatov that was posed in a survey by Comfort, Hoffman and Remus.

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