

## Topologies related to (I)-envelopes

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The (I)-envelope of a set  $A$  in a dual Banach space is defined by

$$(\text{I-} \text{env}(A) = \bigcap \left\{ \overline{\bigcup_{n \in \mathbb{N}} \text{conv } A_n}^{w^* \|\cdot\|} : A = \bigcup_{n \in \mathbb{N}} A_n \right\}.$$

This notion, inspired by the notion of (I)-generation from [2], was introduced in [3]. It was used in [4, 1, 5], in particular to characterize Grothendieck property and its quantitative version.  $(\text{I-} \text{env}(A)$  is a norm-closed convex set and  $\overline{\text{conv } A}^{\|\cdot\|} \subset (\text{I-} \text{env}(A) \subset \overline{\text{conv } A}^{w^*}$  for any set  $A$ . We will address the following natural problem:

**Question.** Let  $X$  be a Banach space. Is there a (locally convex) topology  $\tau$  on  $X^*$  such that  $(\text{I-} \text{env}(A) = \overline{\text{conv } A}^\tau$  for each  $A \subset X^*$ ?

The answer to the ‘locally convex’ version is ‘sometimes yes, sometimes no’, but a complete characterization is still missing. The ‘topological’ version is widely open and is connected to several interesting intermediate topologies on  $X^*$ .

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