# On $\sigma$-metacompact function spaces

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We introduce the following property of a family $\mathcal{L}$ of subsets of a set $S$:

(∗) For all $N \in \mathcal{L}$ and $x \in S$, there exists a finite subset $A$ of $N$ such that, for each $L \in \mathcal{L}$, if $x \in L$ and $L \cap N \neq \emptyset$, then $L \cap A \neq \emptyset$.

We consider compact spaces which have a $k$-network with property (∗). Examples of spaces which do not admit a $k$-network with (∗) include $\beta\omega$, a compact scattered space of height $\omega + 1$ and the one-point compactification of a tree-space.

**Theorem.** *If $K$ is a compact space which has a $k$-network with property (∗), then $C_p(K)$ is hereditarily $\sigma$-metacompact.*

*Supercompact spaces* are usually defined by the existence of a “binary” subbase for the closed subsets, but according to a known and easy result, every supercompact space has a binary closed $k$-network.

**Proposition.** *A family $\mathcal{L}$ of compact closed subsets of a space $X$ is binary if, and only if, for all $N \in \mathcal{L}$ and $x \in X$, there exists $a \in N$ such that, for each $L \in \mathcal{L}$, if $x \in L$ and $L \cap N \neq \emptyset$, then $a \in L$.*

Hence every supercompact space has a $k$-network with (∗).

**Corollary.** *$C_p(K)$ is hereditarily $\sigma$-metacompact for every supercompact space $K$.*

**Corollary.** *$C_p(K)$ is hereditarily $\sigma$-metacompact for every dyadic space $K$.*