On dense sets of products of spaces

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The classical Hewitt–Marczewski–Pondiczery theorem states that if $d(X_s) \leqslant \tau$ $(\omega \leqslant \tau)$ for every $s \in S$ and $|S| \leqslant 2^\tau$ then $d(\prod_{s\in S} X_s) \leqslant \tau$.

Very important is the problem of the existence of a dense set of a cardinality τ in the product $\prod_{s\in S} X_s$ which contains no convergent nontrivial sequences.

For $\tau = \omega$ the existence of such set were proved for $I^{\mathfrak{c}}$ (W.H. Priestly, 1970), for $D^{\mathfrak{c}}$, where D is the two point discrete space (P. Simon, 1978), for $Z^{\mathfrak{c}}$, where Z is separable not single point T_1-space (A. Gryzlov, 2018), for a product of $2^{\mathfrak{c}}$ separable decomposable spaces, i.e. spaces, which contain two not empty closed disjoint sets (A. Gryzlov, 2020).

We prove

Theorem. *For a regular cardinal τ the product $\prod_{\alpha\in 2^\tau} X_\alpha$ of decomposable spaces, where $d(X_\alpha) = \tau$ $(\alpha \in 2^\tau)$, contains a dense set Q, $|Q| = \tau$, such that in every subset $P \subseteq Q$, $|P| = \tau$, there is $P' \subseteq P$, $|P'| = \tau$, which contains no convergent nontrivial sequences and therefore is sequentially closed.*

[1] This work carried within the framework of state assigment of Ministry of Science and Higher Education of Russia (FEWS-2020-0009).