A Banach space C(K) reading the dimension of K

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In 2004 Koszmider constructed a compact Hausdorff space K such that whenever L is compact Hausdorff and the Banach spaces of continuous functions $C(K)$ and $C(L)$ are isomorphic, L is not zero-dimensional. We show that, assuming Jensen's diamond principle ($\diamondsuit$), the following strengthening of the above result holds:

Theorem. *Assume $\diamondsuit$. Let $n \in \mathbb{N}$. There is a compact Hausdorff space K, such that if L is compact Hausdorff and $C(K) \sim C(L)$, then the covering dimension of L is equal to n.*

The constructed space is a modification of Koszmider's example. It is a separable connected compact space with the property that every linear bounded operator $T : C(K) \to C(K)$ is a weak multiplication i.e. it is of the form $T(f) = gf + S(f)$, where $g \in C(K)$ and S is a weakly compact operator on $C(K)$.

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