Congruence-free compact semigroups

Roman S. Gigoń

rgigon@ath.bielsko.pl

A classical result of semigroup theory says that a finite *congruence-free* semigroup S (i.e., S has exactly two congruences) without zero such that $\mathsf{card}(S) > 2$ is a simple group. On the other hand, by a *topological* congruence ρ on a topological semigroup S we shall mean an algebraic congruence such that the quotient space S/ρ is a topological semigroup with respect to the quotient topology (notice that an algebraic congruence on a compact semigroup is topological if and only if it is closed). Further, a topological semigroup S is *congruence-free* if the set of its topological congruences is equal to $\{1_S, S\times S\}$. Recall that a topological semigroup S is called *metric* if there exists a subinvariant metric m on S (that is, $m(ca, cb) \leq m(a, b)$ and $m(ac, bc) \leq m(a, b)$ for all $a, b, c \in S$) which determines the topology of S. In this talk, I will present a sketch of the proof of the following result:

Theorem *Every infinite congruence-free compact semigroup S is a connected metric Lie group (so all left and right translations of S are isometries) with cardinality $\mathfrak{c}$.*

In group theory, a *simple* Lie group is a connected locally compact non-Abelian Lie group G which does not have nontrivial *connected* normal subgroups. Clearly, the well-known classification of simple Lie groups has nothing to do with the classification of finite simple groups. On the other hand, it is easy to see that a compact group is congruence-free if and only if it does not have nontrivial *closed* normal subgroups.

Problem *Classify all congruence-free compact groups.*

(see the above theorem)

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