Planar absolute retracts and countable structures

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A compact space is said to be an absolute retract (AR) if it is homeomorphic to a retract of the Hilbert cube. In particular, every AR is a locally connected, simply connected continuum. The following lemma shows an interesting property of ARs in the plane.

Lemma. *If $X, Y \subseteq \mathbb{R}^2$ are absolute retracts, then X is homeomorphic to Y if and only if ∂X is homeomorphic to ∂Y.*

A sketch of the proof of this lemma is going to be shown, as well as examples witnessing the importance of the assumption that X and Y are ARs.

This lemma has a nice consequence belonging to the field of invariant descriptive set theory (a discipline studying complexities of equivalence relations on standard Borel spaces).

Theorem. *The homeomorphism equivalence relation on the class of planar absolute retracts is classifiable by countable structures.*

On the other hand, we also have the following theorem.

Theorem. *The homeomorphism equivalence relation on the class of absolute retracts contained in $\mathbb{R}^3$ is not classifiable by countable structures.*

Question. Is the homeomorphism equivalence relation on the class of planar absolute neighborhood retracts classifiable by countable structures?