Homology of Generalized Generalized Configuration Spaces

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We describe an assignment, to each finite simplicial complex S and each topological space X, of a topological space $M(S, X)$. Our construction generalizes the generalized or graph configuration space $M_G(X)$. In particular $M(Cl(G), X) = M_{G^c}(X)$, where G^c stands for the edge-complement of G and $Cl(G)$ is the clique complex of G.

Eastwood–Huggett [1] showed that in case X is a manifold, the Euler characteristic $\chi(M_G(X))$ obeys a deletion-contraction formula, hence $\chi(M_G(\mathbb{CP}^{\lambda-1}))$ is the chromatic polynomial of G evaluated at λ. The characteristic $\chi(M(S, X))$ obeys several similar deletion-contraction formulæ, leading to:

Theorem *If X is a manifold, $\chi(M(S, X))$ is a monic polynomial in $\chi(X)$, whose degree is the number of vertices of S.*

We may thus regard $\chi(M(S, X))$ as a kind of "X-chromatic polynomial" for the simplicial complex S. As in [2], $H_*(M(S, X))$ is an algebraic categorification of this polynomial, and $M(S, X)$ is a topological realization of that categorifcation.

[1] M. Eastwood and S. Huggett, *Euler characteristics and chromatic polynomials*, European J. Combin. **28** (2007), 1553–1560

[2] V. Baranovsky and R. Sazdanović, *Graph homology and graph configuration spaces*, J. Homotopy Rel. Str. **7** (2012), 223–235

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