A cardinality bound for T_2 spaces

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For a space X we introduce the cardinal invariant $aL'(X)$, a weakening of the Lindelöf degree $L(X)$, and establish that $|X| \leq 2^{aL'(X)\chi(X)}$ for any T_2 space X. It can be shown that a) $aL'(X) = \aleph_0$ for an H-closed space X, and b) $aL(X) \leq aL'(X) \leq aL_c(X) \leq L(X)$ for a general T_2 space X. Thus, this result gives a common proof that $|X| \leq 2^{\chi(X)}$ for both the class of Lindelöf spaces (Arhangel'skii) and the class of H-closed spaces ([1]). Recall that a space X is H-closed if every open cover of X has a finite subfamily with dense union.

A set $A \subseteq X$ is said to be E-closed if its absolute, EA, is equal to A. Then $aL'(X)$ is defined as the least cardinal κ such that for every cover $\mathcal{U}$ of an E-closed set A by sets open in X there exists $\mathcal{V} \in [\mathcal{U}]^{\leq\kappa}$ such that $A \subset \bigcup_{V\in\mathcal{V}} \overline{V}$.

To put the established bound in context by recall that

1. $|X| \leq 2^{aL_c(X)t(X)\psi_c(X)}$ for a T_2 space X ([2]);
2. $2^{aL(X)\chi(X)}$ is not a bound for the cardinality of all T_2 spaces ([3]);
3. $aL_c(X)$ is not necessarily countable for H-closed X, as witnessed by the Katětov extension $\kappa\omega$ of the countable discrete space ω.

[1] A. Dow and J. Porter *Cardinalities of H-closed spaces*, in *Proceedings of the 1982 Topology Conference* (1982), pp. 27–50.
[2] A. Bella and F. Cammaroto, *On the cardinality of Urysohn spaces*, Canad. Math. Bull. **31** (1988), no. 2, 153–158