

# A proof of the Tree Alternative Conjecture for the Topological Minor Relation

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The Tree Alternative Conjecture states that the equivalence class of any tree (rooted or unrooted) under mutual embeddability is either 1 or infinite. We prove the analogous for the topological minor relation.

**Theorem.** *For any tree  $T$*

- (1)  $|T| = 1$  or  $|T| \geq \aleph_0$ , and
- (2) for any  $r \in V(T)$ ,  $|(T, r)| = 1$  or  $|(T, r)| \geq \aleph_0$ .

The above is proved by means of stratifying all trees into two complementary categories: those containing all *large* and those containing *small* trees. We then establish the following

**Theorem.** *For any large tree  $T$ ,*

- (1)  $|T| \geq 2^{\aleph_0}$  and
- (2) for any  $r \in V(T)$ ,  $|(T, r)| \geq 2^{\aleph_0}$ .

**Theorem.** *For any small tree  $T$ ,*

- (1)  $|T| = 1$  or  $|T| \geq \aleph_0$ , and
- (2) for any  $r \in V(T)$ ,  $|(T, r)| = 1$  or  $|(T, r)| \geq \aleph_0$ .