

Partitioning the real line into Borel sets

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For which infinite cardinals κ is there a partition of the real line \mathbb{R} into precisely κ Borel sets?

Hausdorff famously proved that there is a partition of \mathbb{R} into \aleph_1 Borel sets. The main theorem presented in this talk is that, other than Hausdorff's result, the spectrum of possible sizes of partitions of \mathbb{R} into Borel sets can be fairly arbitrary. For example, given any $A \subseteq \omega$ with $0, 1 \in A$, there is a forcing extension in which $A = \{n : \text{there is a partition of } \mathbb{R} \text{ into } \aleph_n \text{ Borel sets}\}$.

We also look at the corresponding question for partitions of \mathbb{R} into closed sets. We show that, like with partitions into Borel sets, the set of all uncountable κ such that there is a partition of \mathbb{R} into precisely κ closed sets can be fairly arbitrary.