

# Hereditarily indecomposable continua as Fraïssé limits

Adam Bartoš<sup>\*1</sup>, Wiesław Kubiś

bartos@math.cas.cz,

kubis@math.cas.cz

In 2006, Irwin and Solecki introduced projective Fraïssé theory of topological structures and showed that a pseudo-arc is the Fraïssé limit of the class of all finite linear graphs and quotient maps. They also characterized the pseudo-arc as the unique arc-like continuum  $\mathbb{P}$  such that for every arc-like continuum  $Y$ , every  $\varepsilon > 0$ , and every continuous surjections  $f, g: \mathbb{P} \rightarrow Y$  there is a homeomorphism  $h: \mathbb{P} \rightarrow \mathbb{P}$  such that  $\sup_{x \in \mathbb{P}} d(f(x), g(h(x))) < \varepsilon$ .

We consider an approximate framework for Fraïssé theory where the pseudo-arc itself is the Fraïssé limit of the category  $\mathcal{I}$  of all continuous surjections of the unit interval, in the category  $\sigma\mathcal{I}$  of all arc-like continua and all continuous surjections. The characterizing condition above becomes the *projective homogeneity* condition in our framework.

Similarly, we may consider the category  $\mathcal{S}$  of all continuous surjections of the unit circle, and the category  $\sigma\mathcal{S}$  of all circle-like continua and all continuous surjections. It turns out there is no Fraïssé limit of  $\mathcal{S}$  in  $\sigma\mathcal{S}$ . However, if we restrict to the subcategory  $\mathcal{S}_P \subseteq \mathcal{S}$  of the maps whose degree uses only primes from a fixed set  $P$ , and the subcategory  $\sigma\mathcal{S}_P \subseteq \sigma\mathcal{S}$  of circle-like continua that are limits of inverse sequences of  $\mathcal{S}_P$ -maps, with maps that can be approximated by  $\mathcal{S}_P$ -maps as morphisms, then the corresponding Fraïssé limit is the  $P$ -adic pseudo-solenoid  $\mathbb{P}_P$ , and it is characterized as the unique  $\sigma\mathcal{S}_P$ -object that is *projectively homogeneous*, or equivalently has the *projective extension property*.

---

<sup>1</sup>The authors were supported by GA ČR (Czech Science Foundation) grant EXPRO 20-31529X and RVO: 67985840.