Tree sums and maximal connected I-spaces

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If $\mathcal{P}$ is a property of topological spaces, we say that a topological space $\langle X, \tau \rangle$ (or the corresponding topology τ) is *maximal* $\mathcal{P}$ if it has the property $\mathcal{P}$, but no strictly finer topology $\tau^* > \tau$ has the property $\mathcal{P}$. We are interested in the case where $\mathcal{P}$ means connectedness, i.e. in *maximal connected topologies*.

We show how the property of being maximal connected (and also *essentially connected* and *strongly connected*) is preserved by the construction of so-called *tree sums* of topological spaces under certain conditions.

We also recall the characterization of *finitely generated* maximal connected spaces and reformulate it in the language of *specialization preorder* and graphs, from which it is imminent that finitely generated maximal connected spaces are precisely certain tree sums of copies of the Siepiński space.

Finally, we note that several classes of topological spaces are equivalent to the class of *I-spaces* in the realm of maximal connected spaces, and that maximal connected I-spaces include the class of finitely generated maximal connected spaces. We present several results towards characterization of maximal connected I-spaces.

[1] The author was supported by the grant SVV-2016-260336 of Charles University in Prague and by the University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics (MathMAC).

TOPOSYM Prague 2016