

# On completeness and topologizability of countable semigroups

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In this talk we discuss a connection between categorical closedness and topologizability of semigroups. For a class  $\mathbf{T}_1\mathbf{S}$  of  $T_1$  topological semigroups we show that a countable semigroup  $X$  with finite-to-one shifts is injectively  $\mathbf{T}_1\mathbf{S}$ -closed if and only if  $X$  is  $\mathbf{T}_1\mathbf{S}$ -nontopologizable in the sense that every  $T_1$  semigroup topology on  $X$  is discrete. Moreover, a countable cancellative semigroup  $X$  is absolutely  $\mathbf{T}_1\mathbf{S}$ -closed if and only if every homomorphic image of  $X$  is  $\mathbf{T}_1\mathbf{S}$ -nontopologizable. Also, we discuss a notion of a polybounded semigroup. It is proved that a countable semigroup  $X$  with finite-to-one shifts is polybounded if and only if  $X$  is  $\mathbf{T}_1\mathbf{S}$ -closed if and only if  $X$  is  $\mathbf{T}_z\mathbf{S}$ -closed, where  $\mathbf{T}_z\mathbf{S}$  is a class of Tychonoff zero-dimensional topological semigroups. We show that polyboundedness provides an automatic continuity of the inversion in  $T_1$  paratopological groups and prove that every cancellative polybounded semigroup is a group.

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