Automatic continuity of measurable homomorphisms on Čech-complete topological groups

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The problem of automatic continuity of measurable homomorphisms traces its history back to Cauchy who proved in 1821 that continuous additive real functions are linear and asked about conditions implying the continuity of additive functions. In the very first issue of Fundamenta Mathematice (1920), three papers (of Banach, Sierpiński and Steinhaus) were dedicated to the proof of the continuity of Lebesgue measurable additive real functions. Later those results were extended by Weil, Pettis, Christensen and other great mathematicians of XX century who substantially contributed to the theory of automatic continuity. In the talk we discuss the resent progress in extending classical results on automatic continuity beyond the class of Polish groups. In particular, we present a new

Theorem. *A homomorphism $h : X \to Y$ from a Čech-complete topological group X to a topological group Y is continuous iff h is Borel-measurable iff h is universally measurable iff h is universally BP-measurable. If X is ω-narrow (and locally compact), then h is continuous iff h is BP-measurable (iff h is Haar-measurable).*

This theorem extends a recent result of Rosendal (2019) on the continuity of universally measurable homomorphisms between Polish groups, and an old result of Kleppner (1989–91) on the continuity of Haar-measurable homomorphisms between locally compact groups. Details can be seen at (`arxiv.org/abs/2206.02481`).