Hyperspaces of Euclidean spaces in the Gromov–Hausdorff metric

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The Gromov–Hausdorff distance d_{GH} is a useful tool for studying topological properties of families of metric spaces. For two compact metric spaces X and Y the number $d_{GH}(X,Y)$ is defined to be the infimum of all Hausdorff distances $d_H(i(X),j(Y))$ for all metric spaces M and all isometric embeddings $i: X \to M$ and $j: Y \to M$.

Clearly, the Gromov–Hausdorff distance between isometric spaces is zero; it is a metric on the family GH of isometry classes of compact metric spaces. The metric space $(\mathrm{GH}, \mathrm{d_{GH}})$ is called the Gromov–Hausdorff hyperspace.

This talk is devoted to the subspace $\mathrm{GH}(\mathbb{R}^n)$ of GH consisting of the classes $[E] \in$GH whose representative E is a metric subspace of the Euclidean space $\mathbb{R}^n$, $n \geq 1$. $\mathrm{GH}(\mathbb{R}^n)$ is called the Gromov–Hausdorff hyperspace of $\mathbb{R}^n$. One of the main results of this talk asserts that $\mathrm{GH}(\mathbb{R}^n)$ is homeomorphic to the orbit space $2^{\mathbb{R}^n}/E(n)$, where $2^{\mathbb{R}^n}$ is the hyperspace of all nonempty compact subsets of $\mathbb{R}^n$ endowed with the Hausdorff metric and $E(n)$ is the isometry group of $\mathbb{R}^n$. This is applied to prove that $\mathrm{GH}(\mathbb{R}^n)$ is homeomorphic to the Hilbert cube with a removed point.

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