Companions of partially ordered sets

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The results we discuss were motivated by the (now retracted) claim by N. Howes and W. Sconyers that "every normal, linearly Lindelöf space is Lindelöf." Let $(D, \leq)$ be a partially ordered set. A well ordered set $(C, \preceq)$ is called a companion of $(D, \leq)$ provided C is a cofinal subsets of $(D, \leq)$, and $\preceq$ is a well order on C such that for every $c_1, c_2 \in C$ if $c_1 \leq c_2$ then $c_1 \preceq c_2$. The Ordering Lemma says that every partially ordered set has a companion. Given a directed set $(D, \leq)$ and a net $f : D \to X$, the restriction $f \upharpoonright C$ of the net to the companion is a transfinite sequence. We discuss how the convergence and clustering of $f \upharpoonright C$ is related to the convergence and clustering of f. We discuss the difference between converging and clustering of transfinite sequences. We give an example to show that a companion sequence $f \upharpoonright C$ can have a cluster point but f has no cluster point.