

# A local Ramsey theory for block sequences

Iian B. Smythe<sup>1</sup>

ismythe@math.cornell.edu

Gowers [1] proved an approximate Ramsey theorem for analytic partitions of the space of block sequences in a Banach space. Exact, discretized, versions of this result were later given by Rosendal [2]. We isolate the combinatorial properties of the space of block sequences which enable these constructions, and prove that they can be carried out within certain subfamilies, analogous to selective coideals and the role they play in Mathias' [3] local form of Silver's theorem for analytic partitions of  $[\mathbb{N}]^\infty$ . We consider applications of these results to understanding the combinatorial structure of projections in Calkin algebra. Under large cardinal assumptions, these results are extended to partitions in  $L(\mathbb{R})$ .

- [1] W. Gowers, *An infinite ramsey theorem and some banach-space dichotomies*, *Annals of Mathematics* (2) **156** (2002), no. 3, 797-833
- [2] C. Rosendal, *An exact ramsey principle for block sequences*, *Collectanea Mathematica* **61** (2010), no. 1, 25-36
- [3] A. Mathias, *Happy families*, *Annals of Mathematical Logic* **12** (1977a), no. 1, 59-111

Copyright © Smythe

---

<sup>1</sup> The author has been partially supported by NSERC and the NSF

