Characterizing Noetherian spaces as Δ^0_2-analogue to compact spaces

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In the presence of suitable power spaces, compactness of $\mathbf{X}$ can be characterized as the singleton $\{\emptyset\}$ being open in $\mathcal{A}(\mathbf{X})$. Equivalently, this means that universal quantification over a compact space preserves open predicates.

Using the language of represented spaces, one can make sense of notions such as a Σ^0_2-subset of the space of Σ^0_2-subsets of a given space [1]. This suggests higher-order analogues to compactness: We can, e.g. , investigate the spaces $\mathbf{X}$ where $\{\emptyset\}$ is a Δ^0_2-subset of the space of Δ^0_2-subsets of $\mathbf{X}$. Call this notion Δ^0_2-compactness. As Δ^0_2 is self-dual, we find that both universal and existential quantifier over Δ^0_2-compact spaces preserve open predicates.

Recall that a space is called Noetherian iff every subset is compact. Within the setting of Quasi-Polish spaces [2], we can fully characterize the Δ^0_2-compact spaces. Note that the restriction to Quasi-Polish spaces is sufficiently general to include plenty of examples.

Theorem *A Quasi-Polish space is Noetherian iff it is Δ^0_2-compact.*

[1] A. Pauly and M. de Brecht, *Towards synthetic descriptive set theory: An instantiation with represented spaces*, arXiv 1307.1850.

[2] M. de Brecht, *Quasi-Polish spaces*, Annals of Pure and Applied Logic **164** (2013), no. 3, 354–381

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