Minimal homeomorphisms of a Cantor space: full groups and invariant measures.

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A homeomorphism g of a Cantor space X is said to be *minimal* when all g-orbits are dense. Its *full group* $[g]$, the group of all homeomorphisms of X which map each g-orbit onto itself, is intimately related to the orbit partition induced by g: two minimal homeomorphisms g, h whose full groups are isomorphic (as abstract groups) are *orbit equivalent*, that is, there exists a homeomorphism φ of X such that the g-orbit of any $x \in X$ is mapped by φ onto the h-orbit of $\varphi(x)$. This is analogous to what happens in the measure-preserving context; in that case, full groups admit a useful Polish group topology. We prove that such is not the case in the topological context and that full groups of minimal homeomorphisms are coanalytic non-Borel subsets of $\mathrm{Homeo}(X)$. We then argue that the *closure* of $[g]$ inside $\mathrm{Homeo}(X)$ is an interesting object of study. Denoting by K_g the set of all g-invariant Borel probability measures on X, a combination of results of Glasner–Weiss and Giordano–Putnam–Skau shows that $\overline{[g]} = \{h \in \mathrm{Homeo}(X) : \forall \mu \in K_g \; h_*\mu = \mu\}$ and that $\overline{[g]}$ is again a complete invariant for orbit equivalence. The question arises of characterizing which sets K of probability measures on X can be realized as the set of all invariant measures for some minimal homeomorphism g. I will describe an answer to that question, extending a result of Akin (corresponding to the case when K is a singleton) and running parallel to some unpublished work of Dahl.

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[1] Both authors were partially supported by ANR grant Grupoloco.

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