History, structure, results and problems on hyperspaces and symmetric products

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Given a space X there are several ways to construct a new space $K(X)$ from X. Given a continuum X (compact, connected, metric space) we consider the hyperspaces

1. $2^X = \{A \subset X : A$ is nonempty and closed $\}$;
2. $C(X) = \{A \in 2^X : A$ is connected $\}$;
3. $C_n(X) = \{A \in 2^X : A$ has at most n components $\}$; and
4. $F_n(X) = \{A \in 2^X : A$ has at most n points $\}$

Note that $C(X) = C_1(X)$ and $F_1(X)$ is homeomorphic to X.

A continumm X is a cone provided that there exists a space Z such that X is homeomorphic to the cone of Z. Given a hyperspace $K(X) \in \{2^X, C_n(X), F_n(X)\}$ there are several natural problems in the sructure of Hyperspaces. We discuss three in this talk.

Problem *For which continua X is the hyperspace $K(X)$ a cone.*

Problem *Can the space X be recovered when we know the hyperspace $K(X)$.*

Problem *Determine the homogeneity degree of a hyperspace $K(X)$.*

In this talk we establish the history of these problems, survey what has been done in this direction for the past five years and set several problems still unsolved.

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