$\mathfrak{G}$-bases in free objects over uniform spaces

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Denote by ω^ω the set of natural sequences endowed with the partial order: $f \le g$ iff $f(n) \le g(n)$ for all $n \in \omega$. We say that a uniform space X has a $\mathfrak{G}$-*base* if its uniformity $\mathcal{U}(X)$ admits a base of entourages $(U_\alpha)_{\alpha\in\omega^\omega}$ such that $U_\beta \subset U_\alpha$ for all elements $\alpha \le \beta$ in ω^ω. By $C_u(X)$ we denote the space of all uniformly continuous real-valued functions on X endowed with the pointwise partial order.

Theorem *The free locally convex space $L_u(X)$ of a uniform space X has a local $\mathfrak{G}$-base if and only if the uniformity $\mathcal{U}(X)$ of X has a $\mathfrak{G}$-base and the poset $C_u(X)$ is ω^ω-dominated.*

Similar sufficient conditions are found which imply that the free linear topological space $V_u(X)$ of a uniform space X has a local $\mathfrak{G}$-base.

Theorem *If the free locally convex space $L(X)$ of a k-space X has a local $\mathfrak{G}$-base, then the double function space with the compact-open topology $C_k(C_k(X))$ has a local $\mathfrak{G}$-base, too.*

The talk is based on the results of preprints [1], [2], [3].

[1] A. Leiderman, V. Pestov, and A. Tomita, *On topological groups admitting a base at identity indexed with ω^ω*, preprint (2015)
[2] T. Banakh and A. Leiderman, *$\mathfrak{G}$-bases in free (locally convex) linear topological spaces*, preprint (2016a)
[3] T. Banakh and A. Leiderman, *$\mathfrak{G}$-bases in free (Abelian) topological groups*, preprint (2016b)

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