On κ-metrizable spaces

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The concept of a κ-metrizable spaces was introduced by E. Shchepin in 1976. Let $RC(X)$ denote the set of all regular closed sets of a topological space X. A topological space X is *κ-metrizable* if there exists a function $\rho : X \times RC(X) \to [0, \infty)$ satisfying the following conditions:

1. $\rho(x, C) = 0$ if and only if $x \in C$ for every $x \in X$,

2. If $C \subseteq D$, then $\rho(x, C) \geq \rho(x, D)$ for every $x \in X$,

3. $\rho(\cdot, C)$ is a continuous function for every $x \in X$,

4. $\rho(x, cl(\bigcup_{\alpha<\lambda} C_\alpha)) = \inf_{\alpha<\lambda} \rho(x, C_\alpha)$ for every non-decreasing totally ordered sequence $\{C_\alpha : \alpha < \lambda\} \subset RC(X)$ and every $x \in X$.

We say that ρ is *κ-metric* if it satisfies condition $(1) - (4)$. If ρ fulfills condition $(1) - (3)$ and $\rho(x, cl(\bigcup_{n<\omega} C_n)) = \inf_{n<\omega} \rho(x, C_n)$ for any chain $\{C_n : n < \omega\} \subset RC(X)$ and any $x \in X$, then we say that ρ is *countable κ-metric*. If a space X has a countable κ-metric, then we call this space *countably κ-metrizable*.

We show that κ-metrizable spaces is a proper subclass of countable κ-metrizable spaces. On the other hand, for pseudocompact spaces the new class coincides with κ-metrizable spaces. We prove a generalization of Chigogidze result that Čech–Stone compactification of pseudocompact countable κ-metrizable space is κ-metrizable. We also give a new characterization of existence measurable cardinal using countable κ-metric.