Baire one functions depending on finitely many coordinates

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Let $(X_n)_{n=1}^{\infty}$ be a sequence of topological spaces, $P = \prod_{n=1}^{\infty} X_n$ and $a = (a_n)_{n\in\mathbb{N}} \in P$ be a point. For every $n \in \mathbb{N}$ and $x \in P$ we put $p_n(x) = (x_1, \dots, x_n, a_{n+1}, a_{n+2}, \dots)$. We say that a set $A \subseteq P$ *depends on finitely many coordinates*, if there exists $n \in \mathbb{N}$ such that for all $x \in A$ and $y \in P$ the equality $p_n(x) = p_n(y)$ implies $y \in A$. A map $f : X \to Y$ defined on a subspace $X \subseteq P$ is *finitely determined* if it *depends on finitely many coordinates*, i.e., $f(x) = f(y)$ for all $x, y \in X$ with $p_n(x) = p_n(y)$. We denote by $\mathrm{CF}(X, Y)$ the set of all continuous finitely determined maps between X and Y; we write $\mathrm{CF}(X)$ for $\mathrm{CF}(X, \mathbb{R})$.

We answer two questions from [1] and prove, in particular, that every Baire one function on a subspace of a countable perfectly normal product is the pointwise limit of a sequence of continuous functions, each depending on finitely many coordinates. It is proved also that a lower semicontinuous function on a subspace of a countable perfectly normal product is the pointwise limit of an increasing sequence of continuous functions, each depending on finitely many coordinates, if and only if the function has a minorant which depends on finitely many coordinates.

[1] V. Bykov, *On Baire class one functions on a product space*, Topology and its Applications **199** (2016), no. 1-3, 55–62